Oscillations

Question1

A particle executes simple harmonic motion with an amplitude of 4cm. At the mean position, velocity of the particle is 10cm/ s. The distance of the particle from the mean position when its speed becomes 5cm/ s is √α cm, where α = ___

[27-Jan-2024 Shift 1]

Answer: 12

Solution:

$$V_{\text{at mean position}} = A\omega \Rightarrow 10 = 4\omega$$

$$\omega = \frac{5}{2}$$

$$V = \omega\sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$

$$x = \sqrt{12}\text{ cm}$$

Question2

A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be :

[27-Jan-2024 Shift 2]

Options:

A.

$\tan^{-1}(\sqrt{2})$

B.

$2\tan^{-1}(1/2)$

C.

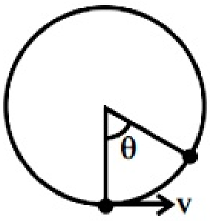
$\tan^{-1}(1/2)$

D.

$2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Answer: B

Solution:



Loss in kinetic energy = Gain in potential energy

$\Rightarrow \frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$

$\Rightarrow \frac{v^2}{\ell} = 2g(1-\cos\theta)$

Acceleration at lowest point $= -\frac{v^2}{\ell}$

Acceleration at extreme point $= g\sin\theta$

Hence, $\frac{v^2}{\ell} = g\sin\theta$

$\therefore \sin\theta = 2(1-\cos\theta)$

$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$

Question3

When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is x/8, where x =____

[29-Jan-2024 Shift 1]

Answer: 9

Solution:

Let total energy $= E = \frac{1}{2}KA^2$

$$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2 \times 9} = \frac{E}{9}$$

$$KE = E - \frac{E}{9} = \frac{8E}{9}$$

$$\text{Ratio } \frac{\text{Total}}{\text{KE}} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

$$X = 9$$

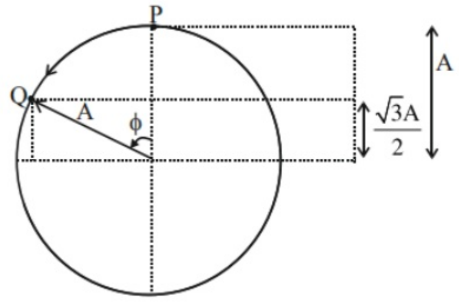
Question4

A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from x = A to x = √3/2 A will be π/x s, where x = _____:

[29-Jan-2024 Shift 2]

Answer: 2

Solution:



From phasor diagram particle has to move from P to Q in a circle of radius equal to amplitude of SHM.

$$\cos\phi = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6}$$

Now, $\frac{\pi}{6} = \omega t$

$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

$$\frac{\pi}{6} = \frac{2\pi}{6\pi}t$$

$$t = \frac{\pi}{2}$$

So, $x = 2$

Question5

A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is 4m, then the time period of small oscillations will be ____s. [. take $g = \pi^2\ ms^{-2}$]

[30-Jan-2024 Shift 2]

Answer: 8

Solution:

Acceleration due to gravity $g' = \frac{g}{4}$

$$T = 2\pi\sqrt{\frac{4\ell}{g}}$$

$$T = 2\pi\sqrt{\frac{4\times 4}{g}}$$

$$T = 2\pi\frac{4}{\pi} = 8\ s$$

Question6

A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is 2A/3. The new amplitude of motion is nA/3. The value of n is___

[31-Jan-2024 Shift 1]

Answer: 7

Solution:

$$v = \omega\sqrt{A^2 - x^2}$$

at $x = \frac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}A\omega}{3}$$

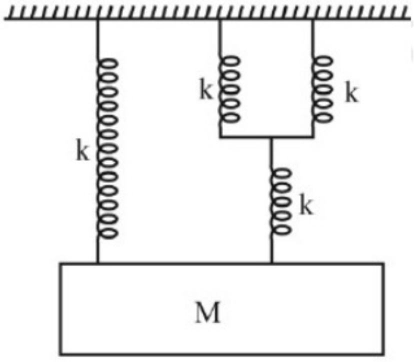
New amplitude $= A'$

$$v' = 3v = \sqrt{5}A\omega = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$$

$$A' = \frac{7A}{3}$$

Question7

The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5K}}$, where the value of α is _____



[31-Jan-2024 Shift 2]

Answer: 12

Solution:

$$k_{eq} = \frac{2k \cdot k}{3k} + k = \frac{5k}{3}$$

Angular frequency of oscillation $(\omega) = \sqrt{\frac{k_{eq}}{m}}$

$$(\omega) = \sqrt{\frac{5k}{3m}}$$

Period of oscillation $(\tau) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3m}{5k}}$

$$= \pi\sqrt{\frac{12m}{5k}}$$

Question8

A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillations if a mass 9m is suspended from the same spring is f_2. The value of f_1/f_2 is:

[1-Feb-2024 Shift 2]

Answer: 3

Solution:

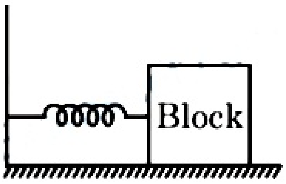
$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

Question9

For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg the angular frequency is ω_2. The ratio ω_2 / ω_1 is :



[30-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 2

D. $\frac{1}{2}$

Answer: B

Solution:

Solution:

$\omega = \sqrt{\frac{k}{m}}$

$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$

Question10

The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}$ s. The value of x is ________.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

[30-Jan-2023 Shift 2]

Answer: 88

Solution:

Solution:

$4v^2 = 50 - x^2$

$\Rightarrow v = \frac{1}{2}\sqrt{50 - x^2}$

$\omega = \frac{1}{2}$

$T = \frac{2\pi}{\omega} = 4\pi = \frac{88}{7}$

$x = 88$

Question11

The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At A /2, the kinetic energy

of the block is
[31-Jan-2023 Shift 1]

Options:

A. 37.5J

B. 9.75J

C. 18.75J

D. 12.5J

Answer: C

Solution:

Solution:

$u_{max} = \frac{1}{2}m\omega^2A^2 = 25J$

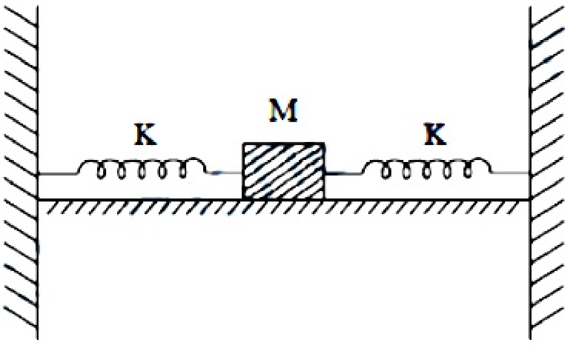
KE at $\frac{A}{2} = \frac{1}{2}mv_1^2 = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$KE = \frac{1}{2}m\omega^2\frac{3A^2}{4} = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

$KE = \frac{3}{4} \times 25 = 18.75J$

Question12

In the figure given below. a block of mass M = 490g placed on a frictionless table is connected with two springs having same spring constant ($K = 2Nm^{-1}$). If the block is horizontally displaced through ' X 'm then the number of complete oscillations it will make in 14π seconds will be _______

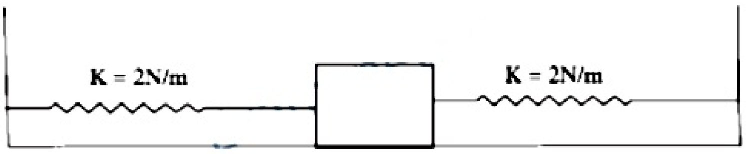


[31-Jan-2023 Shift 1]

Answer: None

Solution:

Solution:



Keff = K + K as both springs are in use in parallel
$= 2k$
$= 2 \times 2 = 4N/m \quad m = 490\,gm$
$= 0.49\,kg$

$T = 2\pi\sqrt{\frac{m}{Keff}} = 2\pi\sqrt{\frac{0.49\,kg}{4}}$

$= 2\pi\sqrt{\frac{49}{400}} = 2\pi\frac{7}{20} = \frac{7\pi}{10}$

No. of oscillation in the 14π is

$N = \frac{time}{T} = \frac{14\pi}{7\pi/10} = 20$

Ans in 20 .

Question13

A particle executes simple harmonic motion between x = −A and x = +A. If time taken by particle to go from x = 0 to $\frac{A}{2}$ is 2 s; then time taken by particle in going from x = $\frac{A}{2}$ to A is :
[25-Jan-2023 Shift 2]

Options:

A. 3 s

B. 2 s

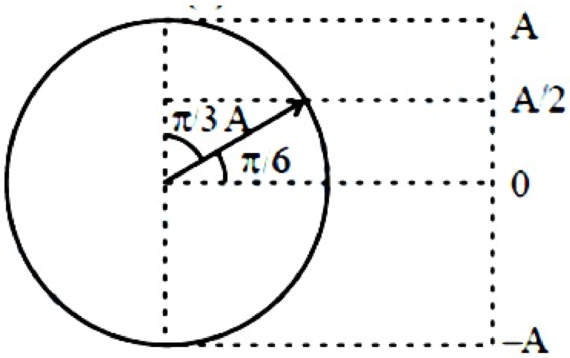
C. 1.5 s

D. 4 s

Answer: D

Solution:

Solution:



Let time from 0 to A / 2 is t_1
& from A / 2 to A is t_2
then $\omega t_1 = \pi/6$
$\omega t_2 = \pi/3$
$\frac{t_1}{t_2} = \frac{1}{2}$
$t_2 = 2t_1 = 2 \times 2 = 4$ sec

Question14

A particle of mass 250g executes a simple harmonic motion under a periodic force $F = (-25x)$ N. The particle attains a maximum speed of 4m / s during its oscillation. The amplitude of the motion is ________ cm. [29-Jan-2023 Shift 2]

Answer: 40

Solution:

Solution:

$\frac{1}{4}a = -25x;\ a = -100x$

$\omega^2 = 100\quad \omega = 10,$

$\omega A = 4\quad A = \frac{4}{10} = 0.4m$

$A = 40\,cm$

Question15

The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is _______.
[30-Jan-2023 Shift 1]

Answer: 8

Solution:

Solution:

$x = A\sin(\omega t)$

$U_{(x)} = \frac{1}{2}kx^2$

$\frac{dU}{dt} = \frac{1}{2}k2x\frac{dx}{dt}$

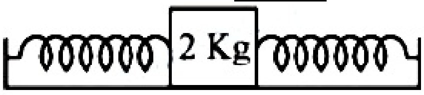
$= kA^2\omega\sin\omega t\cos\omega t \times \frac{2}{2}$

$\left(\frac{dU}{dt}\right)_{max} = \frac{kA^2\omega}{2}(\sin 2\omega t)_{max}$

$2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4}\omega = \frac{T}{8} \Rightarrow \beta = 8$

Question16

A block of mass 2 kg is attached with two identical springs of spring constant 20N / m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is_____



[24-Jan-2023 Shift 1]

Answer: 5

Solution:

Solution:

$F = -2kx,\ a = -\frac{2kx}{m},\ \omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \times 20}{2}}$

$= \sqrt{20}\ \text{rad / s}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$

$x = 5$

Question17

A body of mass 200g is tied to a spring of spring constant 12.5N / m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad / s, then the ratio of extension in the spring to its natural length will be :
[24-Jan-2023 Shift 2]

Options:

A. 1 : 2

B. 1 : 1

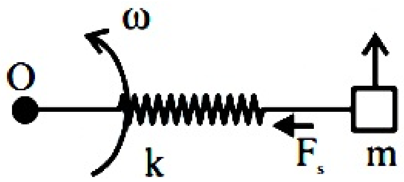
C. 2 : 3

D. 2 : 5

Answer: C

Solution:

Solution:



Natural length $= L_0$

Extension $= x$

$kx = m(L_0 + x)\omega^2$

$\Rightarrow 12.5x = \frac{1}{5}(L_0 + x)25 \Rightarrow 1.5x = L_0$

$\Rightarrow \frac{x}{L_0} = \frac{2}{3}$

Question18

A mass m attached to free end of a spring executes SHM with a period of 1 s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass m is ___kg.
[24-Jan-2023 Shift 2]

Answer: 1

Solution:

Solution:

$T = 2\pi\sqrt{\frac{m}{k}} = 1$

$T' = 2\pi\sqrt{\frac{m+3}{k}} = 2$

$\frac{T}{T'} = \sqrt{\frac{m}{m+3}} = \frac{1}{2}$

$\Rightarrow \frac{m}{m+3} = \frac{1}{4}$

$m = 1$

Question19

The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) - time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is _______.
[30-Jan-2023 Shift 1]

Answer: 8

Solution:

$x = A\sin(\omega t)$

$U_{(x)} = \frac{1}{2}kx^2$

$\frac{dU}{dt} = \frac{1}{2}k2x\frac{dx}{dt}$

$= kA^2\omega \sin\omega t \cos\omega t \times \frac{2}{2}$

$\left(\frac{dU}{dt}\right)_{max} = \frac{kA^2\omega}{2}(\sin 2\omega t)_{max}$

$2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4}\omega = \frac{T}{8} \Rightarrow \beta = 8$

Question20

The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is:_______ cm.
[1-Feb-2023 Shift 1]

Answer: 2

Solution:

$KE = PE + \frac{PE}{4}$

$KE = \frac{5}{4}PE$

$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{5}{4} \times \frac{1}{2}m\omega^2x^2$

$\left[v = \omega\sqrt{A^2 - x^2}\right]$

$A^2 - x^2 = \frac{5}{4}x^2$

$\frac{9x^2}{4} = A^2$

$x = \frac{2}{3}A$

$\therefore x = \frac{2}{3} \times 3\,cm$

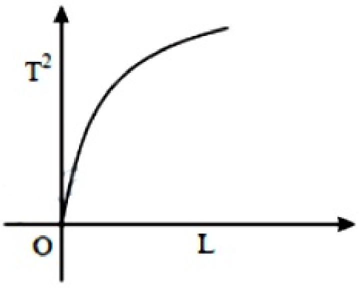
$x = 2\,cm$

Question21

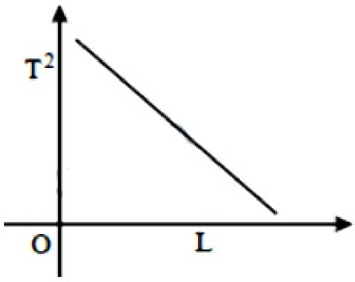
Choose the correct length (L) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.
[1-Feb-2023 Shift 2]

Options:

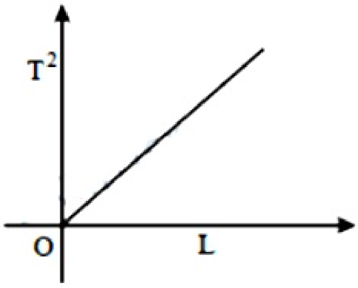
A.



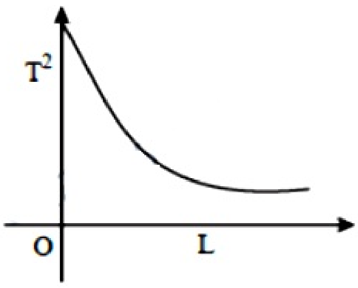
B.



C.



D.



Answer: C

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = \frac{4\pi^2}{g} \times \ell$$

$$T^2 \alpha \ell$$

Question22

A simple pendulum with length 100 cm and bob of mass 250g is executing S.H.M. of amplitude 10 cm. The maximum tension in the

string is found to be $\frac{x}{40}$N. The value of x is ______.
[6-Apr-2023 shift 2]

Answer: 99

Solution:

Solution:
For pendulum

$T_{max} = mg + \frac{mv^2}{L} \ldots (1)$

Given $m = \frac{1}{4}kg, L = 1m, g = 9.8m / s^2$

and amplitude $A = \frac{1}{10}m$

For SHM, $KE_{max} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2$

using $\omega = \sqrt{\frac{g}{L}}$

$mv^2 = m\left(\sqrt{\frac{g}{L}}\right)^2 A^2 = \frac{mgA}{L} \ldots (2)$

using (2) in (1)

$T_{max} = 2\,mg + \frac{mgA^2}{L^2}$

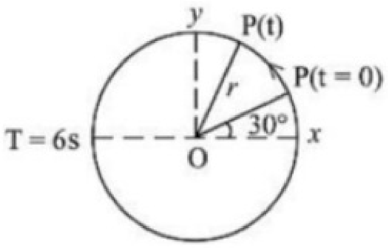
$= mg\left[1 + \frac{1}{10^2}\right] = \frac{1}{4} \times 9.8 \times \frac{101}{100}$

or $T_{max} = \frac{98.98}{40}$

Therefore $x = 99$

Question23

For particle P revolving round the centre O with radius of circular path r and angular velocity ω, as shown in below figure, the projection of OP on the x-axis at time t is



[8-Apr-2023 shift 2]

Options:

A. $x(t) = r\cos\left(\omega t + \frac{\pi}{6}\right)$

B. $x(t) = r\cos\left(\omega t - \frac{\pi}{6}\omega\right)$

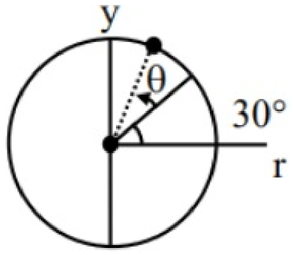
C. $x(t) = r\cos(\omega t)$

D. $x(t) = r\sin\left(\omega t + \frac{\pi}{6}\right)$

Answer: A

Solution:

Solution:
$\theta = \omega t$



Angle from x axis $= \omega t + \frac{\pi}{6}$

Projection of OP on x axis $= r\cos\left(\omega t + \frac{\pi}{6}\right)$

Question24

A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positives x-axis. The displacement of particle in time t is $x = A\sin(\omega t + \delta)$, then the value δ will be
[10-Apr-2023 shift 1]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: D

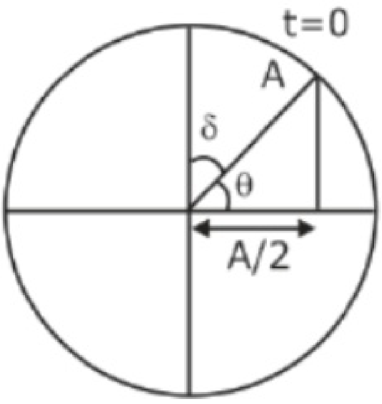
Solution:

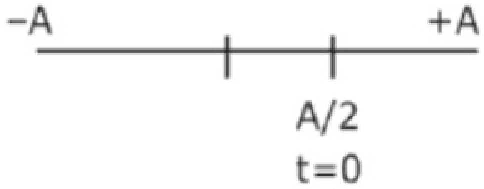
Solution:

$\cos\theta = \frac{A}{2 \times A} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\delta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$





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Question25

A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1m and time period 3.14 s. The maximum force exerted by spring on block is _______ N.
[10-Apr-2023 shift 2]

Answer: 20

Solution:

Solution:
When an object executes S.H.M, its morning acceleration is given by $a_{max} = \omega^2 A$

Where $\omega = \frac{2\pi}{T}$

Therefore, $a_{max} = \frac{4\pi^2 A}{T^2}$

(Max force) $F_{max} = ma_{max} = 5 \times \frac{4 \times 3.14 \times 3.14}{3.14 \times 3.14} \times (1)$

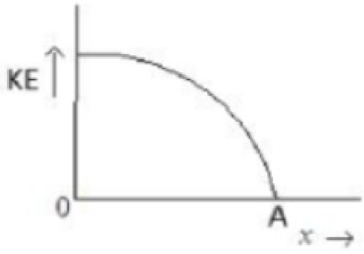
$= 20N$

Question26

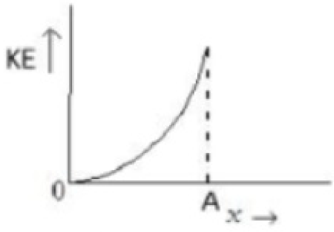
The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by
[11-Apr-2023 shift 1]

Options:

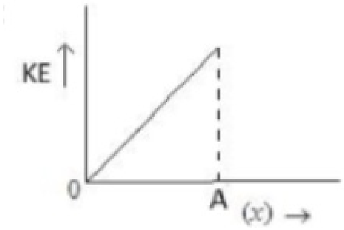
A.



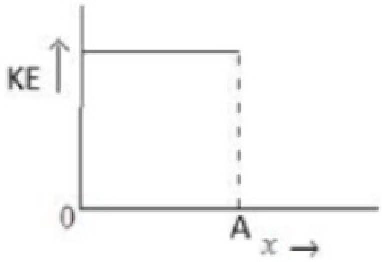
B.



C.



D.



Answer: A

Solution:

Solution:

$K \cdot E = T \cdot E - P \cdot E$

$K \cdot E = \frac{1}{2} KA2 - \frac{1}{2} Kx^2$

Graph b / w$K \cdot E$ and x will be parabola

Option → (1)

Question27

A particle is executing simple harmonic motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be
[12-Apr-2023 shift 1]

Options:

A. 1 : 1

B. 1 : 3

C. 1 : 4

D. 2 : 1

Answer: B

Solution:

Solution:
Ratio of PE and KE = ?

When $\underset{x\backslash\ disp.}{x} = \frac{A}{2}$

$KE = \frac{1}{2}m\omega^2[A^2 - x^2]$

$= \frac{1}{2}m\omega^2\left[A^2 - \frac{A^2}{4}\right]$

$KE = = \frac{1}{2}m\omega^2\left[\frac{3A^2}{4}\right]$ ... (1)

And $PE = \frac{1}{2}m\omega^2x^2$

$= \frac{1}{2}m\omega^2\frac{A^2}{4}$

$\frac{PE}{KE} = \frac{m\omega^2A^2 \times 8}{8m\omega^2A^2 \times 3} = \frac{1}{3}$

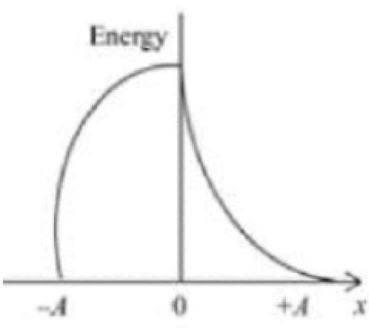
$\Rightarrow 1 : 3$

Question28

Which graph represents the difference between total energy and potential energy of a particle executing SHM vs it's distance from mean position?
[13-Apr-2023 shift 1]

Options:

A.



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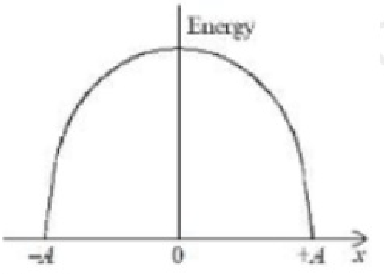
B.



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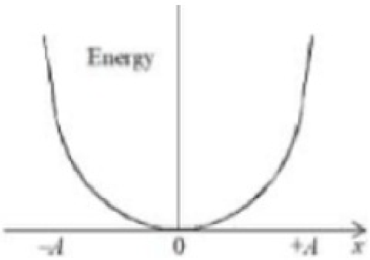

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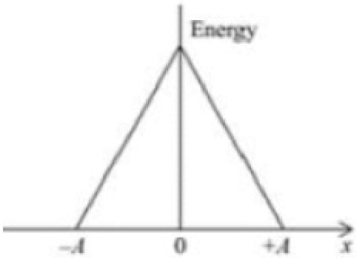
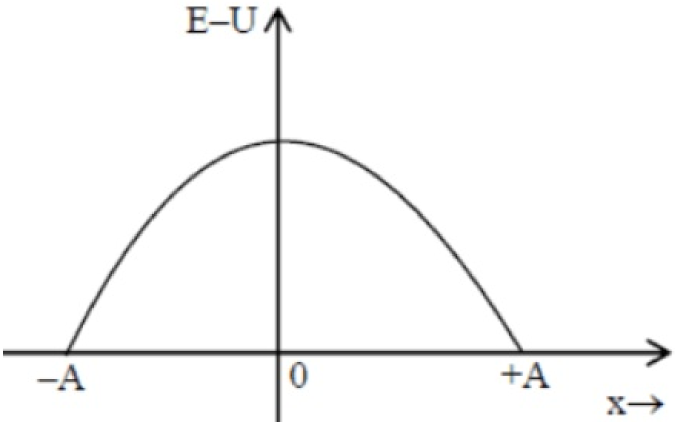


Answer: B

Solution:

Solution:
Total energy in SHM = E
$E = K + U$
$E - U = K$
$E - U = \frac{1}{2}m\omega^2(A^2 - x^2)$



Question29

At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A\cos(30^\circ)$. If amplitude is 40 cm and kinetic

energy at that time is 200J, the value of force constant is $1.0 \times 10^{x} Nm^{-1}$. The value of x is ______
[13-Apr-2023 shift 1]

Answer: 4

Solution:

$v = \omega\sqrt{A^2 - x^2}$

$y = A \times \frac{\sqrt{3}}{2}$

$v = \omega\sqrt{A^2 - \frac{3A^2}{4}} = \frac{\omega A}{2}$

Given, KE = 200J

$\frac{1}{2}m\frac{\omega^2 A^2}{4} = 200$

$KA^2 = 1600 \ (K = m\omega^2)$

$K = \frac{1600}{(40 \times 10^{-2})^2}$

$K = 10^4 N/m$

$x = 4$

Question30

A particle executes SHM of amplitude A. The distance from the mean position when its's kinetic energy becomes equal to its potential energy is :
[13-Apr-2023 shift 2]

Options:

A. $\sqrt{2A}$

B. $\frac{1}{2}A$

C. $\frac{1}{\sqrt{2}}A$

D. 2A

Answer: C

Solution:

Solution:

K.E = P.E

$\frac{1}{2}M\omega^2(A^2 - x^2) = \frac{1}{2}M\omega^2 x^2$

$(A^2 - x^2) = x^2$

$2x^2 = A^2$

$x = \pm \frac{A}{\sqrt{2}}$

Question31

In a linear Simple Harmonic Motion (SHM)
(A) Restoring force is directly proportional to the displacement.
(B) The acceleration and displacement are opposite in direction.
(C) The velocity is maximum at mean position.
(D) The acceleration is minimum at extreme points.
Choose the correct answer from the options given below :
[15-Apr-2023 shift 1]

Options:

A. (C) and (D) only

B. (A), (C) and (D) only

C. (A), (B) and (C) only

D. (A), (B) and (D) only

Answer: C

Solution:

In SHM,
$F \propto -x \rightarrow A$
$a \propto -x \rightarrow B$
$V_{mean} \rightarrow \text{max imum} \rightarrow c$
$a_{extreme} \rightarrow \text{maximum}$
Hence, (A), (B) and (C) are true.

Question32

Two massless springs with spring constants 2k and 9k, carry 50g and 100g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitudes will be:
[24-Jun-2022-Shift-2]

Options:

A. 1 : 2

B. 3 : 2

C. 3 : 1

D. 2 : 3

Answer: B

Solution:

Solution:

$\omega_1 A_1 = \omega_2 A_2$

$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}$

$= \sqrt{\frac{k_2}{m_2}} \times \sqrt{\frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$

Question33

The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :
[27-Jun-2022-Shift-1]

Options:

A. 6s

B. 8s

C. 12s

D. 36s

Answer: D

Solution:

Solution:

Time taken by the harmonic oscillator to move from mean position to half of amplitude is $\frac{T}{12}$

So, $\frac{T}{12} = 3$

$T = 36$ sec.

Question34

The equation of a particle executing simple harmonic motion is given by $x = \sin \pi \left(t + \frac{1}{3} \right)$ m. At t = 1s, the speed of particle will be (Given : $\pi = 3.14$)
[27-Jun-2022-Shift-2]

Options:

A. $0 \, cm \, s^{-1}$

B. $157 \, cm \, s^{-1}$

C. $272 \, cm \, s^{-1}$

D. $314 \mathrm{~cm} \mathrm{~s}^{-1}$

Answer: B

Solution:

Solution:

$x=\sin\left(\pi t+\frac{\pi}{3}\right) m$

$\Rightarrow \frac{dx}{dt}=\pi \cos\left(\pi t+\frac{\pi}{3}\right)$

$=\pi \cos\left(\pi+\frac{\pi}{3}\right)$ at $t=1 s$

$=-\frac{\pi}{2} m / s$

or $\left|\frac{dx}{dt}\right|=157 \mathrm{~cm} / s$

Question35

A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is s.
[27-Jun-2022-Shift-2]

Answer: 1

Solution:

$A=8 \mathrm{~cm}$
$T=6 \mathrm{~s}$
$A \cos\left(\frac{2\pi t}{T}\right)=\frac{A}{2}$
$\Rightarrow \frac{2\pi t}{T}=\frac{\pi}{3}$
or $t=\frac{T}{6}=1 s$

Question36

A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is____
[29-Jun-2022-Shift-1]

Answer: 700

Solution:

Solution:

$v = \omega\sqrt{A^2 - y^2}$

$\Rightarrow 3\omega\sqrt{10^2 - 5^2} = \omega\sqrt{(A')^2 - 5^2}$

$\Rightarrow 9 \times 75 = (A')^2 - 25$

$\Rightarrow A' = \sqrt{28 \times 25}$ cm

$\Rightarrow x = 700$

Question37

The motion of a simple pendulum executing S.H.M. is represented by the following equation.
$y = A\sin(\pi t + \varphi)$, where time is measured in second. The length of pendulum is
[29-Jun-2022-Shift-2]

Options:

A. 97.23 cm

B. 25.3 cm

C. 99.4 cm

D. 406.1 cm

Answer: C

Solution:

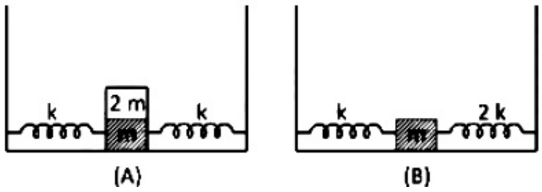
Solution:

$\omega = \pi = \sqrt{\frac{g}{l}}$

So, $l = \frac{g}{\pi^2}$

$\simeq 99.4$ cm (Nearest value)

Question38



In figure (A), mass '2 m' is fixed on mass ' m ' which is attached to two springs of spring constant k. In figure (B), mass ' m ' is attached to two

springs of spring constant ' k ' and ' 2k'. If mass ' m ' in (A) and in (B) are displaced by distance ' x ' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation.
[25-Jul-2022-Shift-1]

Options:

A. $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

B. $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

C. $\frac{T_1}{T_2} = \sqrt{\frac{2}{3}}$

D. $\frac{T_1}{T_2} - \frac{\sqrt{2}}{3}$

Answer: A

Solution:

Solution:
Both the springs are in parallel combination in both the diagrams so
$T_1 = 2\pi\sqrt{\frac{3m}{2k}}$
and $T_2 = 2\pi\sqrt{\frac{m}{3k}}$
So, $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

Question39

The length of a seconds pendulum at a height h = 2R from earth surface will be:
(Given R = Radius of earth and acceleration due to gravity at the surface of earth, $g = \pi^2 ms - 2$)
[25-Jul-2022-Shift-2]

Options:

A. $\frac{2}{9}m$

B. $\frac{4}{9}m$

C. $\frac{8}{9}m$

D. $\frac{1}{9}m$

Answer: D

Solution:

Solution:

$g = \frac{GM}{(R+h)^2} = \frac{GM}{9R^2} = \frac{g_0}{9}$

$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{\frac{g_0}{9}}}$

$\Rightarrow 2 = 2\pi\sqrt{\frac{9l}{g_0}}$

$\Rightarrow l = \frac{g_0}{9\pi^2} = \frac{1}{9}m$

Question40

Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is ____(degree). [25-Jul-2022-Shift-2]

Answer: 60

Solution:

$A_{net} = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1A_2\cos\varphi}$

$\sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\varphi}$

$3A^2 = 2A^2 + 2A^2\cos\varphi$

$\cos\varphi = \frac{1}{2}$

$\varphi = 60^\circ$

Question41

When a particle executes Simple Hormonic Motion, the nature of graph of velocity as a function of displacement will be :
[26-Jul-2022-Shift-1]

Options:

A. Circular

B. Elliptical

C. Sinusoidal

D. Straight line

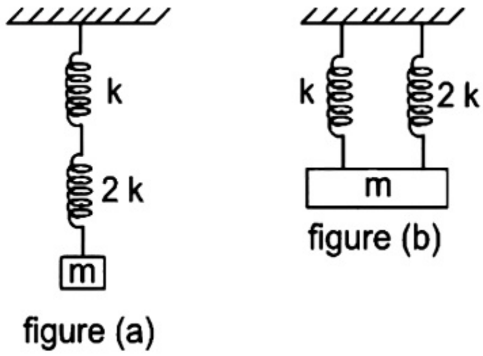
Answer: B

Solution:

Let $x = A \sin \omega t$
$\Rightarrow v = A\omega \cos \omega t$
$\Rightarrow v = \pm \omega \sqrt{A^2 - x^2}$
$\Rightarrow \frac{v^2}{\omega^2} + x^2 = A^2$
$\Rightarrow$ Ellipse

Question42

As per given figures, two springs of spring constants k and 2k are connected to mass m. If the period of oscillation in figure (a) is 3s, then the period of oscillation in figure (b) will be $\sqrt{x}$s. The value of x is ________.



[26-Jul-2022-Shift-2]

Answer: 2

Solution:

For figure (a) :
$K_{eq} = \frac{K \times 2K}{K + 2K} = \frac{2K}{3}$
$T = 2\pi \sqrt{\frac{m}{K_{eq}}} = 2\pi \sqrt{\frac{m}{2K/3}} = 2\pi \sqrt{\frac{3m}{2K}}$
For figure (b):
$K_{eq} = 3K, T' = 2\pi \sqrt{\frac{m}{3K}}$
$\frac{T'}{T} = \sqrt{\frac{m \times 2K}{3K \times 3m}} = \frac{\sqrt{2}}{3}$
$T' = \sqrt{2}$
$x = 2$

Question43

Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q_0 with mass ' m ' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be :
[27-Jul-2022-Shift-1]

Options:

A. $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

B. $\sqrt{\frac{q_0 Q}{4\pi^3\varepsilon_0 ma^3}}$

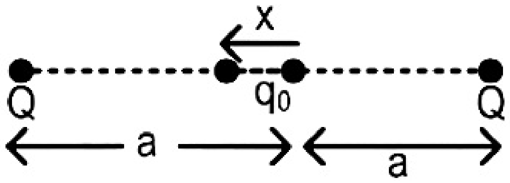
C. $\sqrt{\frac{2\pi^2\varepsilon_0 ma^3}{q_0 Q}}$

D. $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

Answer: A

Solution:

Solution:



(x< < a)(α is acceleration)

$F_{net} = -\left(\frac{kq_0Q}{(a-x)^2} - \frac{kQq_0}{(a+x)^2}\right)$

$m\alpha = -\frac{kq_0Q}{a^4}4ax$

$\Rightarrow \alpha = -\frac{4kq_0Q}{ma^3}x$

So, $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{4q_0Q}}$

or $T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0Q}}$

Question44

Two bar magnets oscillate in a horizontal plane in earth's magnetic field

with time periods of 3s and 4s respectively. If their moments of inertia are in the ratio of 3 : 2, then the ratio of their magnetic moments will be:
[27-Jul-2022-Shift-1]

Options:

A. 2 : 1

B. 8 : 3

C. 1 : 3

D. 27 : 16

Answer: B

Solution:

Solution:

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{I_1}{M_1B_H}}}{2\pi\sqrt{\frac{I_2}{M_2B_H}}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}\times\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{3}{2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\frac{3}{2}\times\frac{M_2}{M_1} = \frac{9}{16}$$

$$\frac{M_1}{M_2} = \frac{8}{3}$$

Question45

A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha-1}$, then the value of α will be ________.
[27-Jul-2022-Shift-1]

Answer: 16

Solution:

$\frac{1}{2}kA^2 = \frac{p^2}{2m}$

$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = \frac{m_2}{m_1} = \frac{1024}{900}$

$\Rightarrow \frac{A_1}{A_2} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1}$

$\therefore \alpha = 16$

Question46

A compass needle of oscillation magnetometer oscillates 20 times per minute at a place P of dip 30°. The number of oscillations per minute become 10 at another place Q of 60° dip. The ratio of the total magnetic field at the two places ($B_Q : B_P$) is :
[27-Jul-2022-Shift-2]

Options:

A. $\sqrt{3} : 4$

B. $4 : \sqrt{3}$

C. $\sqrt{3} : 2$

D. $2 : \sqrt{3}$

Answer: A

Solution:

Solution:

$T = 2\pi\sqrt{\frac{I}{B_H M}}$

$T_1 = 3 \text{ sec} = 2\pi\sqrt{\frac{I}{(B_P \cos 30^\circ)M}}$

$T_2 = 6 \text{ sec} = 2\pi\sqrt{\frac{I}{(B_Q \cos 60^\circ)M}}$

$\frac{3}{6} = \sqrt{\frac{1}{\left(B_P \frac{\sqrt{3}}{2}\right)} \times (B_Q/2)}$

$\frac{3}{6} = \sqrt{\left(\frac{B_Q}{\sqrt{3}B_P}\right)}$

$\frac{\sqrt{3}}{4} = \frac{B_Q}{B_P}$

$B_Q : B_P = \sqrt{3} : 4$

Question47

The potential energy of a particle of mass 4 kg in motion along the x-axis is given by $U = 4(1 - \cos 4x)$J. The time period of the particle for

small oscillation (sinθ ≃ θ) is $\left(\frac{\pi}{K}\right)$ s. The value of K is ________.
[28-Jul-2022-Shift-2]

Answer: 2

Solution:

Solution:
$U = 4(1 - \cos 4x)$
$F = -\frac{dU}{dx} = -4(+\sin 4x)4 = -16\sin(4x)$
For small θ
$\sin\theta \approx \theta$
$F = -64x$
$a = -64x / m = -16x$
$\omega^2 = 16$
$T = \frac{2\pi}{\omega} = \frac{\pi}{2}$

Question48

The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by :
[29-Jul-2022-Shift-1]

Options:

A. $2\pi\sqrt{L / (g\cos\alpha)}$

B. $2\pi\sqrt{L / (g\sin\alpha)}$

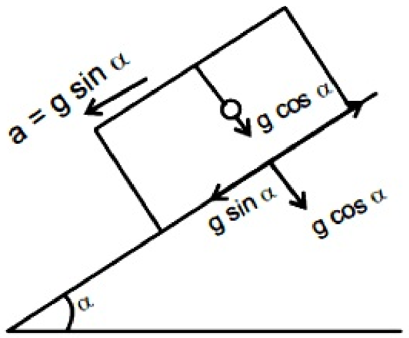
C. $2\pi\sqrt{L / g}$

D. $2\pi\sqrt{L / (g\tan\alpha)}$

Answer: A

Solution:

Solution:
$g_{cff} = g\cos\alpha$



$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$

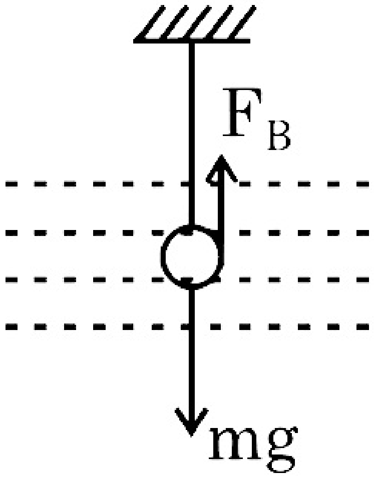
Question49

The metallic bob of simple pendulum has the relative density 5 . The time period of this pendulum is 10s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$s. The value of x will be ______.
[29-Jul-2022-Shift-2]

Answer: 5

Solution:

Solution:



$mg' = mg - F_B$

$g' = \frac{mg - F_B}{F_B}$

$= \frac{\rho_B Vg - \rho_w Vg}{\rho_B V}$

$= \left(\frac{\rho_B - \rho_w}{\rho_B}\right) g$

$= \frac{5-1}{5} \times g$

$= \frac{4}{5} g$

We know, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{\frac{4}{5}g}} = \sqrt{\frac{5}{4}}$$

$$T' = T\sqrt{\frac{5}{4}} = \frac{10}{2}\sqrt{5}$$

$$T' = 5\sqrt{5}$$

Question50

When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is [24 Feb 2021 Shift 2]

Options:

A. circular

B. elliptical

C. parabolic

D. straight line

Answer: B

Solution:

Solution:
Since, the particle is executing SHM. Therefore, displacement equation of wave will be

$y = A\sin\omega t$

$\Rightarrow y / A = \sin\omega t$

and wave velocity equation will be

$$v_y = \frac{dy}{dt} = A\omega\cos\omega t$$

$\Rightarrow v_y / A\omega = \cos\omega t$

Now, $\sin^2\omega t + \cos^2\omega t = 1$

$\therefore (y / A)^2 + (v_y / A\omega)^2 = 1$

This equation is similar to the equation of ellipse.

Question51

Given below are two statements:
Statement I A second's pendulum has a time period of 1s.
Statement II It takes precisely one second to move between the two extreme positions.
In the light of the above statements, choose the correct answer from the options given below.
[26 Feb 2021 Shift 2]

Options:

A. Both Statement I and Statement II are false.

B. Statement I is false but Statement II is true.

C. Statement I is true but Statement II is false.

D. Both Statement I and Statement II are true.

Answer: B

Solution:

Solution:
Statement I is false because time period of second's pendulum is always 2s.
Therefore, time taken to move between two extreme positions will be $T/2 = 2/2 = 1s$
Hence, option (b) is the correct.

Question52

Time period of a simple pendulum is T. The time taken to complete 5 / 8 oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is .... [26 Feb 2021 Shift 2]

Answer: 7

Solution:

Solution:

Given, angular displacement to complete $\frac{5}{8}\left(=\frac{1}{2}+\frac{1}{8}\right)$ rev

$= \left(\pi + \frac{\pi}{6}\right) \text{rad}$

$= \left(\frac{7\pi}{6}\right) \text{rad}$

Since, $\omega = \frac{2\pi}{T} = \frac{\theta}{t}$

$\Rightarrow \theta = \frac{2\pi}{T} \cdot t \Rightarrow \frac{7\pi}{6} = \frac{2\pi}{T} t$

$\Rightarrow t = \frac{7T}{12} = \frac{\alpha}{\beta} T$

Hence, $\alpha = 7$

Question53

If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor [26 Feb 2021 Shift 1]

Options:

A. $\frac{1}{2}, \sqrt{2}$

B. $\frac{1}{4}, \sqrt{2}$

C. $\frac{1}{4}, 2\sqrt{2}$

D. $\frac{1}{2}, 2\sqrt{2}$

Answer: A

Solution:

Solution:
Let series equivalent of spring constant $= k_{eq}$ and T be the time period.

In series arrangement, $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

$\Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_1} = \frac{2}{k_1} \Rightarrow k_{eq} = \frac{k_1}{2}$

As, $T = 2\pi\sqrt{\frac{m}{k_1}}$

where, m is mass of body connected with spring.

$\Rightarrow T \propto \sqrt{\frac{1}{k_1}}$

and $T' \propto \sqrt{\frac{2}{k_1}} \Rightarrow T' = \sqrt{2}T$

Question54

Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $\left(\frac{R}{2}\right)$ from the earth's centre, where R is the radius of the earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period?
[26 Feb 2021 Shift 1]

Options:

A. $\frac{2\pi R}{g}$

B. $\frac{g}{2\pi R}$

C. $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

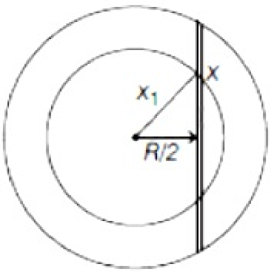
D. $2\pi\sqrt{\frac{R}{g}}$

Answer: D

Solution:

Solution:
Given, radius of earth $= R$

Distance of chord from centre of earth $= \frac{R}{2}$



Let x_1 be the radius of inner circle and M be the mass of earth.

$\therefore m'$(effective mass of earth) $= \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi {x_1}^3$

$\Rightarrow m' = \frac{M}{R^3} {x_1}^3$

If F is the gravitational force exerted by earth on particle at position x and ω be the angular velocity in time period T, then

$F = \frac{Gm'm}{{x_1}^2} = \frac{Gm}{{x_1}^2} \cdot \frac{M}{R^3} {x_1}^3$

$\Rightarrow m\omega^2 x_1 = \frac{GM\,mx_1}{R^3}$

$\Rightarrow \omega = \sqrt{\frac{GM}{R^3}}$

$F = \frac{Gm'm}{{x_1}^2} = \frac{Gm}{{x_1}^2} \cdot R^3 {x_1}^3$

$\Rightarrow m\omega^2 x_1 = \frac{GM\,mx_1}{R^3}$

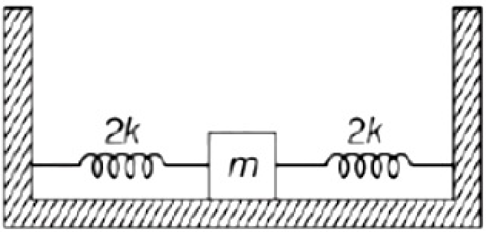
$\Rightarrow \omega = \sqrt{\frac{GM}{R^3}}$

Since, $\omega = \frac{2\pi}{T}$ and $GM = gR^2$

Substituting the above values in Eq. (i), we get 1$\Rightarrow$ $\frac{2\pi}{T} = \sqrt{\frac{gR^2}{R^3}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

Question55

Two identical springs of spring constant 2k are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is



[25 Feb 2021 Shift 2]

Options:

A. $2\pi\sqrt{\frac{m}{2k}}$

B. $2\pi\sqrt{\frac{m}{k}}$

C. $\pi\sqrt{\frac{m}{k}}$

D. $\pi\sqrt{\frac{m}{2k}}$

Answer: C

Solution:

Solution:
Let spring constants of two springs be k_1 and k_2. Since, two springs are connected in parallel connection and parallel equivalent spring constant, $k_{eq} = k_1 + k_2$
$\Rightarrow k_{eq} = 2k + 2k = 4k$
As, time period, $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$
$\Rightarrow T = 2\pi\sqrt{\frac{m}{4k}} = \frac{2\pi}{2}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$

Question56

If the time period of a 2m long simple pendulum is 2s, the acceleration due to gravity at the place, where pendulum is executing SHM is [25 Feb 2021 Shift 1]

Options:

A. $\pi^2 ms^{-2}$

B. $9.8 ms^{-2}$

C. $2\pi^2 ms^{-2}$

D. $16 ms^{-2}$

Answer: C

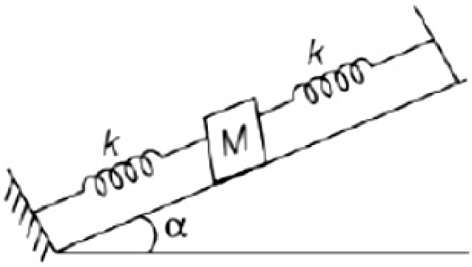
Solution:

Solution:
Given, length of simple pendulum, $I = 2m$
Time period, $T = 2s$
Let g_{eff} be the acceleration due to gravity.
$\therefore$ Time period, $T = 2\pi\sqrt{\frac{1}{g_{eff}}}$
$\Rightarrow g_{eff} = 4\pi^2\frac{1}{T^2}$
$= 4\pi^2\cdot\frac{2}{4} = 2\pi^2 ms^{-2}$

Question57

In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, then the

frequency of oscillation of given body is



[24 Feb 2021 Shift 2]

Options:

A. $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

B. $\frac{1}{2\pi}\sqrt{\frac{2k}{M g \sin\alpha}}$

C. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

D. $\frac{1}{2\pi}\sqrt{\frac{k}{M g \sin\alpha}}$

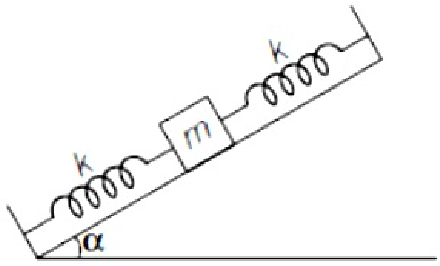
Answer: C

Solution:

Solution:
Let T be the time period of oscillation, then

$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$

$\therefore T = 2\pi\sqrt{\frac{M}{2k}}$ $[\because k_{eq} = k + k]$



and frequency (f) $= \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

Question58

A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin, i.e. $ma = -\alpha x^2$. The distance at which the particle stops is
[24 Feb 2021 Shift 2]

Options:

A. $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{2}}$

B. $\left(\frac{2mv_0}{3\alpha}\right)^{\frac{1}{3}}$

C. $\left(\frac{2mv_0^2}{3\alpha}\right)^{\frac{1}{2}}$

D. $\left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Answer: D

Solution:

Solution:
Given, speed of projection $= v_0$
Damping force, $F = ma = -\alpha x^2$
$\Rightarrow a = -\alpha x^2 / m$
Also, $\Rightarrow v d v = a d x = -\frac{\alpha}{m} x^2 d x$
Integrating both sides, we get
$\int_{v_0}^{v} v d v = \int_0^x -\frac{\alpha}{m} x^2 d x$
$\Rightarrow \left(\frac{v^2}{2}\right)_{v_0}^{0} = -\frac{\alpha}{m}\left(\frac{x^3}{3}\right)_0^x$
$\Rightarrow 0 - v_0^2 / 2 = -\frac{\alpha}{m}\frac{x^3}{3} \Rightarrow x = \left(\frac{3m}{2}\frac{v_0^2}{\alpha}\right)^{1/3}$

Question59

A student is performing the experiment of resonance column. The diameter of the column tube is 6cm. The frequency of the tuning fork is 504Hz. Speed of the sound at the given temperature is 336m / s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is
[25 Feb 2021 Shift 1]

Options:

A. 13cm

B. 16.6cm

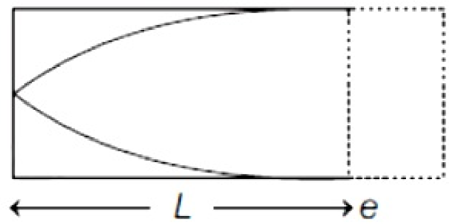
C. 8.4cm

D. 14.8cm

Answer: D

Solution:

Given, diameter of the column tube,
$d = 6cm = 6 \times 10^{-2} m$
Frequency of tuning fork, $f = 504 Hz$ Speed of sound at given temperature, $v = 336 ms^{-1}$
As, this is a closed organ pipe.
Let L be the length of tube and λ be the wavelength, then



and $L + e = \lambda / 4 = v / 4f \quad \left(\because \lambda = \frac{v}{f}\right)$

$$e = 0.6 \times d / 2$$
$$= \frac{6}{10} \times \frac{6 \times 10^{-2}}{2} = 0.018$$
$$\Rightarrow L + e = \frac{v}{4f} = \frac{336}{4 \times 504}$$
$$\Rightarrow L = \frac{336}{2016} - 0.018 = 0.1667 - 0.018$$
$$= 0.1487 m = 14.87 cm$$
$$\sim eq 14.8 cm$$

Question60

A particle executes SHM, the graph of velocity as a function of displacement is
[26 Feb 2021 Shift 2]

Options:

A. a circle

B. a parabola

C. an ellipse

D. a helix

Answer: C

Solution:

Velocity of particle in SHM in terms of displacement x is given as

$$v = \omega \sqrt{A^2 - x^2}$$
$$\Rightarrow v = A\omega \sqrt{1 - x^2 / A^2}$$
$$\Rightarrow \frac{v}{A\omega} = \sqrt{1 - x^2 / A^2}$$
$$\Rightarrow \frac{v^2}{A^2\omega^2} = 1 - \frac{x^2}{A^2}$$
$$\Rightarrow \frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1 \ldots (i)$$

As equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Eq. (i) is similar to the equation of ellipse. So, the graph between velocity and displacement is an ellipse.

Question61

A particle executes SHM with amplitude a and time period T . The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is ....
[26 Feb 2021 Shift 2]

Answer: 3

Solution:

Given, amplitude of particle in SH M $= a$
Maximum velocity, $v = A\omega$
Let displacement be y when speed, $v_2 = v / 2 = \frac{A\omega}{2}$
By using equation of wave velocity,
$v_2{}^2 = \omega^2(A^2 - y^2)$
$\because \left(\frac{\omega A}{2}\right)^2 = \omega^2(A^2 - y^2)$
$\Rightarrow \frac{\omega^2 A^2}{4} = \omega^2(A^2 - y^2)$
$\Rightarrow \frac{A^2}{4} = A^2 - y^2$
$\Rightarrow y = \frac{\sqrt{3}}{2}A$
Hence, $x = 3$

Question62

$Y = A\sin(\omega t + \varphi_0)$ is the time-displacement equation of SHM. At t = 0, the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then, the initial phase angle φ_0 will be
[25 Feb 2021 Shift 2]

Options:

A. $\frac{\pi}{3}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

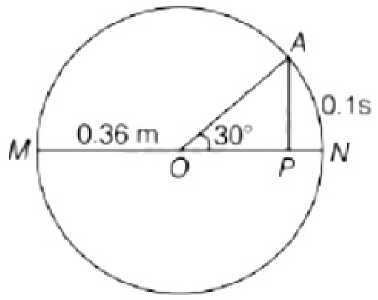
Answer: C

Solution:

Given, displacement-time equation,
$Y = A\sin(\omega t + \varphi_0)$
Here, A is amplitude, ω is angular frequency, t is time taken and φ_0 is the phase constant.
At $t = 0, Y = A / 2$
$\therefore Y = A / 2 = A\sin(0 + \varphi_0) = A\sin\varphi_0$
$\Rightarrow \sin\varphi_0 = 1 / 2$
$\Rightarrow \varphi_0 = \pi / 6$

Question63

The point A moves with a uniform speed along the circumference of a circle of radius 0.36m and covers 30° in 0.1s. The perpendicular projection P from A on the diameter M N represents the simple harmonic motion of P. The restoration force per unit mass when P touches M will be



[25 Feb 2021 Shift 2]

Options:

A. 100 N

B. 9.87 N

C. 50 N

D. 0.49 N

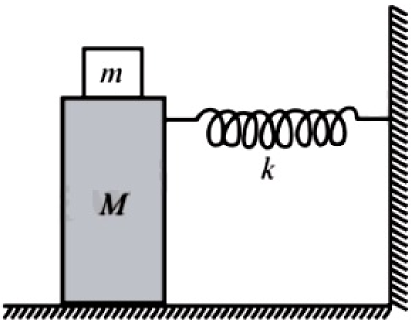
Answer: B

Solution:

Given, radius of circle, $R = 0.36\text{m}$
Angular distance, $\theta = 30^\circ = \pi / 6\text{rad}$
Let / be the arc length.
$\because I = R\theta$
$\Rightarrow I = \frac{36}{100} \times \frac{\pi}{6} = \frac{6\pi}{100}\text{m}$
A s, speed on circular track (v) = $\text{Arclength}(l) / \text{Time}(t)$
$\Rightarrow v = \frac{6\pi}{100 \times 0.1} = \frac{6\pi}{10}\text{ms}^{-1}$
If F be the restoration force and a_t be the radial acceleration ($= v^2 / R$), then
$F = ma_r$
$a_r = \frac{F}{m} = \frac{v^2}{R} = \left(\frac{6\pi}{10}\right)^2 \times \frac{100}{36}$
$= \frac{36 \times 9.87}{100} \times \frac{100}{36} = 9.87\text{N}$

Question64

In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :



[24feb2021shift1]

Options:

A. $A\sqrt{\frac{M-m}{M}}$

B. $A\sqrt{\frac{M}{M+m}}$

C. $A\sqrt{\frac{M+m}{M}}$

D. $A\sqrt{\frac{M}{M-m}}$

Answer: B

Solution:

Solution:
Let the initial velocity of M is V. On putting m on M, let velocity becomes V'.
Momentum of system remains conserved.
$\therefore p_i = p_f \Rightarrow MV = (m+M)V'$
$\Rightarrow MA\omega = (m+M)A\omega (\because V = A\omega)$
$\Rightarrow MA\sqrt{\frac{k}{M}} = (m+M)A'\sqrt{\frac{k}{m+M}}$
$\Rightarrow A' = A\sqrt{\frac{M}{M+m}}$

Question65

Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A

and B is
[17 Mar 2021 Shift 2]

Options:

A. $\frac{k_2}{k_1}$

B. $\frac{k_1}{k_2}$

C. $\sqrt{\frac{k_1}{k_2}}$

D. $\sqrt{\frac{k_2}{k_1}}$

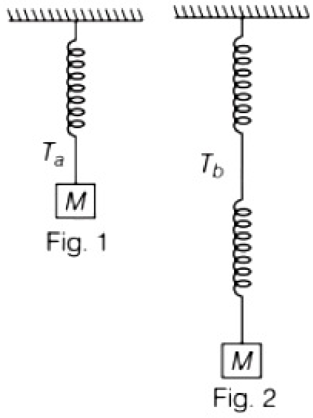
Answer: D

Solution:

Solution:
We know that, the expression of maximum velocity during oscillation,
$V_{max} = A\omega$
Given, $V_{max}(A) = V_{max}(B) \Rightarrow A_1\omega_1 = A_2\omega_2$
Now, the ratio of the amplitude during oscillation,
$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1}$
We know that, $\omega = \sqrt{k/m}$
where, k is the spring constant,
m is the mass of the object.
Substituting the value of ω in Eq. (i), we get $\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

Question66

Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig.2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is ……… (Round off to the nearest integer)



[17 Mar 2021 Shift 1]

Answer: 2

Solution:

Solution:
Time period of Fig. 1 can be given as

$$T_a = 2\pi\sqrt{\frac{M}{k}}$$

where, M is mass of the suspended object and k is the force constant.
In Fig. 2, both the springs are in series combination, therefore its time period can be given as

$$T_b = 2\pi\sqrt{\frac{M}{k_{eq}}} = 2\pi\sqrt{\frac{M}{k/2}} \quad \left(\because k_{eq} = \frac{k \times k}{k + k}\right)$$

Now, $\frac{T_b}{T_a} = \frac{2\pi\sqrt{\frac{M}{k/2}}}{2\pi\sqrt{\frac{M}{k}}}$

$\Rightarrow \frac{T_b}{T_a} = \sqrt{2}$...(i)

According to question, the ratio of time period of oscillation of two SHM is $T_b/T_a = \sqrt{x}$, so on comparing it with Eq. (i) we can say, $x = 2$

Question67

Time period of a simple pendulum is T inside a lift, when the lift is stationary. If the lift moves upwards with an acceleration g/2, then the time period of pendulum will be [16 Mar 2021 Shift 1]

Options:

A. $\sqrt{3}T$

B. $\frac{T}{\sqrt{3}}$

C. $\sqrt{\frac{3}{2}}T$

D. $\sqrt{\frac{2}{3}}T$

Answer: D

Solution:

Solution:
Time period of a simple pendulum can be given as

$T = 2\pi\sqrt{\frac{l}{g}}$...(i)

When the lift moves upwards, then effective acceleration is ⇒

$\Rightarrow g_{eff} = g + a = g + \frac{g}{2} = \frac{3g}{2}$

$\therefore$ New time period, $T_1 = 2\pi\sqrt{\frac{1}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$

$\Rightarrow T_1 = \sqrt{\frac{2}{3}}T$ [From Eq. (i)]

Question68

A block of mass 1kg attached to a spring is made to oscillate with an initial amplitude of 12cm. After 2min, the amplitude decreases to 6cm. Determine the value of the damping constant for this motion.
(Take, In 2 = 0.693)
[17 Mar 2021 Shift 2]

Options:

A. 0.69×10^{2}kg/s

B. 3.3×10^{2}kg/s

C. 1.16×10^{-2}kg/s

D. 5.7×10^{-3}kg/s

Answer: C

Solution:

Solution:
Given, mass block, $m = 1kg$
Initial amplitude, $A_0 = 12cm$
Final amplitude, $A = 6cm$
The time taken to reduce the amplitude, $t = 2min = 120s$
Using the expression of damped oscillation,

$A = A_0 e^{-\frac{b}{2m}t}$

Substituting the values in the above equation, we get

$6 = 12e^{-\frac{b(120)}{2(1)}}$

$\Rightarrow e^{60b} = 2$ or $60b = 10\log 2$

$\Rightarrow b = \frac{0.693}{60} = 1.16 \times 10^{-2}$kg/s

Question69

The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is
[18 Mar 2021 Shift 2]

Options:

A. $\sin(\omega t) = \cos(\omega t)$

B. $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

C. $\sin^2(\omega t)$

D. $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Answer: D

Solution:

Solution:
Time period, $T = \frac{2\pi}{\omega'}$
Given, time period, $T = \frac{\pi}{\omega}$
$\frac{2\pi}{\omega'} = \frac{\pi}{\omega} \Rightarrow \omega' = 2\omega$
Options (a) and (b) are incorrect.
Option (c),
$\sin^2\omega t = \frac{1}{2}(2\sin^2\omega t) = \frac{1}{2}(1 - \cos 2\omega t)$
Hence, the angular frequency is 2ω.
Option (d),
$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$
Angular frequency of SHM is 2ω.
Option (d) is the correct answer.

Question70

A particle performs simple harmonic motion with a period of 2s. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$s. The value of a to the nearest integer is
[18 Mar 2021 Shift 1]

Answer: 6

Solution:

Solution:
Given, the time period of the simple harmonic motion, $T = 2s$
Displacement covered by the particle, $x = A / 2$
Here, A is the amplitude of the particle.
The general equation of the simple harmonic motion,
$x = A\sin\omega t$
$\Rightarrow \frac{A}{2} = A\sin\left(\frac{2\pi}{T}t\right) \Rightarrow \frac{1}{2} = \sin\left(\frac{2\pi}{2}t\right)$
$\Rightarrow \sin\frac{\pi}{6} = \sin(\pi t) \Rightarrow \frac{\pi}{6} = \pi t$
$\Rightarrow t = \frac{1}{6}$
The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{6}$s.
Comparing with $t = \frac{1}{a}$

Hence, the value of the a is 6 .

Question71

For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal?
[17 Mar 2021 Shift 1]

Options:

A. $x = 0$

B. $x = \pm A$

C. $x = \pm \frac{A}{\sqrt{2}}$

D. $x = \frac{A}{2}$

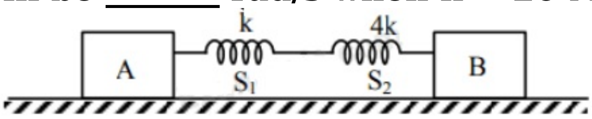
Answer: C

Solution:

Potential energy of SH M $= \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$
where $m =$ mass of particle,
$\omega =$ angular velocity
and $x =$ displacement.
Kinetic energy of SH M $= \frac{1}{2}m\left(\omega\sqrt{A^2 - x^2}\right)^2$
Here, A is the amplitude of SHM.
According to question,
Potential energy of SH M = Kinetic energy of SH M
$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\left(\omega\sqrt{A^2 - x^2}\right)^2$
$\Rightarrow \omega^2x^2 = \omega^2\left(\sqrt{A^2 - x^2}\right)^2 \Rightarrow \omega^2x^2 = \omega^2(A^2 - x^2)$
$\Rightarrow \omega^2x^2 = \omega^2A^2 - \omega^2x^2 \Rightarrow 2\omega^2x^2 = \omega^2A^2$
$\Rightarrow 2x^2 = A^2 \Rightarrow x^2 = \frac{A^2}{2}$
$\Rightarrow x = \pm\sqrt{\frac{A^2}{2}} \Rightarrow x = \pm\frac{A}{\sqrt{2}}$

Question72

In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be _______ rad/s when k = 20 N/m.



[25 Jul 2021 Shift 1]

Answer: 10

Solution:

Solution:

$\omega = \sqrt{\frac{k_{eq}}{\mu}}$

$\mu =$ reduced mass

springs are in series connection

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

$k_{eq} = \frac{k \times 4k}{5k} = \frac{4k}{5}$

$k_{eq} = \frac{4 \times 20}{5} N / m = 16 N / m$

$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{0.2 \times 0.8}{0.2 + 0.8} = 0.16 kg$

$\omega = \sqrt{\frac{16}{0.16}} = \sqrt{100} = 10$

Question73

T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified timeperiod is :
[22 Jul 2021 Shift 2]

Options:

A. T_0

B. $8\pi T_0$

C. $4T_0$

D. $\frac{1}{4}T_0$

Answer: C

Solution:

Solution:

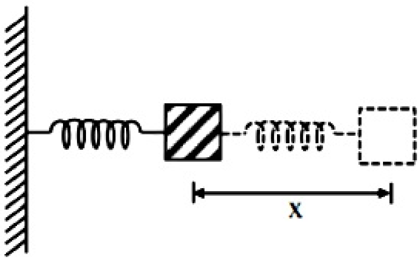
$T_0 = 2\pi\sqrt{\frac{l}{g}}$

New time period $T = 2\pi\sqrt{\frac{l / 16}{g}} = \frac{2\pi}{4}\sqrt{\frac{l}{g}}$

$T = \frac{T_0}{4}$

Question74

The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by x(t) = Asin\ω t+ Bcos\ωtwith $\omega = \sqrt{\frac{K}{m}}$Suppose that at time t = 0, the position of mass is x(0) and velocity v(0), then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where Cand ϕ are :
[22 Jul 2021 Shift 2]

Options:

A. $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

B. $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

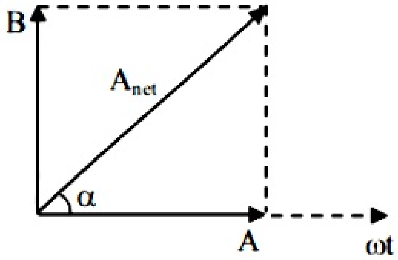
C. $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

D. $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Answer: D

Solution:

Solution:
$x = A\sin\omega t + B\cos\omega t$
$v = \frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$
At t = 0, x(0) = B
v(0) = Aω
$x = A\sin\omega t + B\sin(\omega t + 90°)$



$A_{net} = \sqrt{A^2 + B^2}$
$\tan\alpha = \frac{B}{A} \Rightarrow \cot\alpha = \frac{A}{B}$
$\Rightarrow x = \sqrt{A^2 + B^2}\sin(\omega t + \alpha)$
$\Rightarrow x = \sqrt{A^2 + B^2}\cos(\omega t - (90 - \alpha))$

$x = C\cos(\omega t - \phi)$
$\Rightarrow C = \sqrt{A^2 + B^2}$
$C = \sqrt{\frac{[v(0)]^2}{\omega^2} + [x(0)]^2}$
$\phi = 90 - \alpha$
$\tan\alpha = \cos\alpha = \frac{A}{B}$
$\Rightarrow \tan\phi = \frac{v(0)}{x(0).\omega}$
$\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Question75

A particle executes simple harmonic motion represented by displacement function as
$x(t) = A\sin(\omega t + \phi)$
If the position and velocity of the particle at t = 0s are 2cm and $2\omega cms^{-1}$ respectively, then its amplitude is $x\sqrt{2}$cm where the value of x is ____.
[27 Jul 2021 Shift 2]

Answer: 2

Solution:

Solution:
$x(t) = A\sin(\omega t + \phi)$
$v(t) = A\omega\cos(\omega t + \phi)$
$2 = A\sin\phi$ ....(1)
$2\omega = A\omega\cos\phi$ ....(2)
From (1) and (2) $\tan\phi = 1$
$\phi = 45°$
Putting value of ϕ in equation (1)
$2 = A\left\{\frac{1}{\sqrt{2}}\right\}$
$A = 2\sqrt{2}$
$x = 2$

Question76

A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by:
[27 Jul 2021 Shift 1]

Options:

A. $y = a$

B. $y = \frac{a}{\sqrt{2}}$

C. $y = \frac{a\sqrt{3}}{2}$

D. $y = \frac{a}{2}$

Answer: D

Solution:

Solution:

$E = \frac{1}{2}K a^2$

$\frac{3E}{4} = \frac{1}{2}K(a^2 - y^2)$

$\frac{3}{4} \times \frac{1}{2}K a^2 = \frac{1}{2}K(a^2 - y^2)$

$y^2 = a^2 - \frac{3a^2}{4}$

$y = \frac{a}{2}$

Question77

In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.
[25 Jul 2021 Shift 2]

Options:

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$= \frac{1}{2}m\omega^2\left(\frac{3A^2}{4}\right)$

$K = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

Question78

A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as: [20 Jul 2021 Shift 2]

Options:

A. $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$

B. $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

C. $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$

D. $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

Answer: D

Solution:

Solution:

$v^2 = \omega^2(A^2 - x^2)$

$A^2 = x_1^2 + \frac{v_1^2}{\omega^2} = x_2^2 + \frac{v_2^2}{\omega^2}$

$\omega^2 = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}$

$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

Question79

A particle of mass 1 kg is hanging from a spring of force constant $100 Nm^{-1}$. The mass is pulled slightly downward and released, so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is.

[31 Aug 2021 Shift 1]

Answer: 8

Solution:

Solution:
Given, mass of particle, m = 1kg
Spring constant, $k = 100 Nm^{-1}$
Let time period is T.
Kinetic energy (KE) = Potential energy (PE) = E
Amplitude = A
Angular velocity = ω
Wave displacement = x
KE in SHM = PE in SHM
$\Rightarrow \frac{1}{2} m\omega^2(A^2 - x^2) = \frac{1}{2} kx^2$
$\Rightarrow m\omega^2(A^2 - x^2) = kx^2$ ...(i)
Since, force, $F = m\omega^2 x = kx$
$\Rightarrow k = m\omega^2$
Substituting the value in Eq. (i), we get
$\Rightarrow m\omega^2(A^2 - x^2) = m\omega^2 x^2$
$\Rightarrow A^2 - x^2 = x^2$
$\Rightarrow A^2 = 2x^2$
$\Rightarrow x = \frac{A}{\sqrt{2}}$
Since, wave displacement, $x = A \sin \omega t$
$\Rightarrow \frac{A}{\sqrt{2}} = A \sin \omega t$
$\Rightarrow \sin\left(\frac{\pi}{4}\right) = \sin \omega t$
$\Rightarrow \omega t = \frac{\pi}{4}$
$\Rightarrow \frac{2\pi}{T} . t = \frac{\pi}{4}$
$\Rightarrow t = \frac{T}{8}$
Comparing the given value in the question i.e. $\frac{T}{x}$, the value of x = 8

Question80

A bob of mass m suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by $\frac{1}{3}$rd of the original length, then the time period of the simple harmonic oscillations will be
[31 Aug 2021 Shift 2]

Options:

A. T

B. $\frac{3}{2}$T

C. $\frac{3}{4}$T

D. $\frac{4}{3}$T

Answer: D

Solution:

Solution:
Given that, a bob of mass m suspended by a thread of length l undergoes SHM with time period,

$T_1 = T = 2\pi\sqrt{\frac{l}{g}}$ ...(i)

In liquid, effective gravity,

$g_{eff} = g\left(\frac{\sigma - \rho}{\sigma}\right)$

where, ρ = density of liquid
and σ = density of body (bob).

Given, $\rho = \frac{\sigma}{4}$

$\Rightarrow \sigma = 4\rho$

Substituting this value, we get

$g_{eff} = g\left(\frac{4\rho - \rho}{4\rho}\right) = \frac{3g}{4}$

Now, the new time period of pendulum in liquid is

$T' = 2\pi\sqrt{\frac{l'}{g_{eff}}}$ ...(ii)

Here, $l' = l + \frac{l}{3} = \frac{4l}{3}$

Substituting the values in Eq. (ii), we get

$T' = 2\pi\sqrt{\frac{4l}{3} \times \frac{4}{3g}}$

$= \left(2\pi\sqrt{\frac{l}{g}}\right)\frac{4}{3}$

$\Rightarrow T' = \frac{4T}{3}$ [from Eq. (i)]

Question81

The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass m to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as ....%.
[26 Aug 2021 Shift 2]

Answer: 14

Solution:

Solution:
Given, accuracy of acceleration due to gravity = 4%
Accuracy of time period = 3%
Energy stored in pendulum at any instant is given as

$E = \frac{mgL\theta^2}{2}$ ...(i)

Time period of pendulum is given by expression

$T = 2\pi\sqrt{\frac{L}{g}}$

Rearrange the above expression for L, we get

$L = g\left(\frac{T}{2\pi}\right)^2$ ...(ii)

Substituting the value of L in Eq. (i), we get

$E = \frac{mg^2\theta^2}{2}(T2\pi)^2 = \frac{mg^2\theta^2T^2}{8\pi^2}$

Now, the accuracy in measurement of energy can be calculated as

$\frac{\Delta E}{E} \times 100 = 2\frac{\Delta g}{g} \times 100 + 2\frac{\Delta T}{T} \times 100$ (∵ m,θ and π are constant)

$\Rightarrow \frac{\Delta E}{E} \times 100 = 2 \times 4\% + 2 \times 3\% = 14\%$

Thus, the accuracy to which E is known is 14%.

Question82

For a body executing SHM
A. potential energy is always equal to its kinetic energy.
B. average potential and kinetic energy over any given time interval are always equal.
C. sum of the kinetic and potential energy at any point of time is constant.
D. average kinetic energy in one time period is equal to average potential energy in one time period.
Choose the most appropriate option from the options given below.
[31 Aug 2021 Shift 2]

Options:

A. (C) and (D)

B. Only (C)

C. (B) and (C)

D. Only (B)

Answer: A

Solution:

Solution:
As we know that, for a body executing SHM the average kinetic energy is equal to average potential energy over a complete time period

i.e. $KE_{av} = PE_{av} = \frac{1}{4}m\omega^2a^2$

∴ Total mechanical energy at any time = Sum of kinetic and potential energy $= \frac{1}{2}m\omega^2a^2 =$ constant.

where, m = mass of body executing SHM,
ω = angular frequency
and a = amplitude of SHM.
Hence, (c) and (d) are true.

Question83

Two simple harmonic motion, are represented by the equations $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ Ratio of amplitude of y_1 to

$y_2 = x : 1$. The value of x is ....
[27 Aug 2021 Shift 2]

Answer: 1

Solution:

Solution:

Given, $y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$

$A_1 = 10$

$y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$

$= 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t$

Amplitude of SHM_2 is

$A_2 = \sqrt{5^2 + (5\sqrt{3})^2}$

$= \sqrt{25 + 75}$

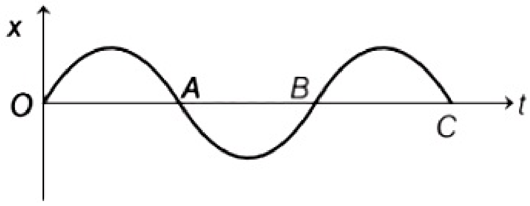
$= \sqrt{100} = 10$

$\therefore$ Ratio of amplitudes, $\frac{A_1}{A_2} = \frac{10}{10} = 1 : 1$

$x = 1$

Question84

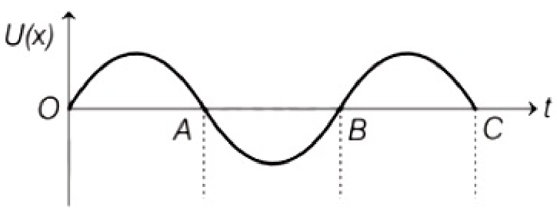
The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.



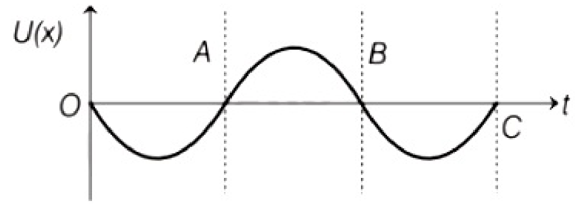
The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :
[27 Aug 2021 Shift 1]

Options:

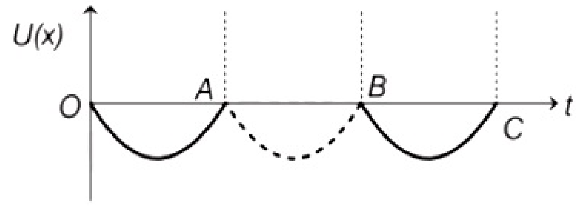
A.



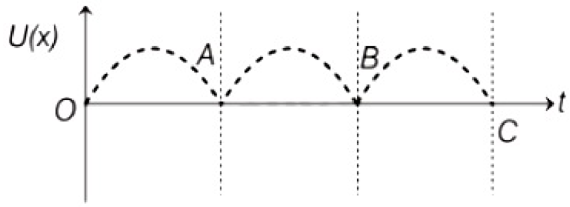
B.



C.



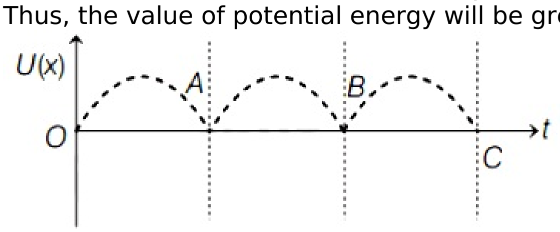
D.



Answer: D

Solution:

Solution:
It is given that motion is free simple harmonic motion.
Consider a spring mass system performing free simple harmonic motion.
Displacement of mass is given as
$x = x_0 \sin \omega t$
The potential energy stored in spring for displacement x can be given as
$PE = \frac{1}{2} kx^2$
Substituting the value of x, we get
$PE = \frac{1}{2} kx_0{}^2 \sin^2 \omega t$
Here, k is spring constant and x_0 is also constant.
$PE = C\sin^2 \omega t \left(\therefore C = \frac{1}{2} kx_0{}^2 \right)$
Simplify the value of $\sin^2$ ωt, we get
$PE = C\left[\frac{1 - \cos 2 \omega t}{2} \right]$
The value of function cos2ωt will always lies between −1 and 1.
Thus, the value of potential energy will be greater than or equal to zero.



$0 \leq PE$
This condition is only satisfies by graph (d).

Question85

Two simple harmonic motions are represented by the equations

$x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and
$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$
The amplitude of second motion is ....times the amplitude in first motion.
[26 Aug 2021 Shift 2]

Answer: 2

Solution:

Solution:
Equation of first simple harmonic motion,
$x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$.
Equation of second simple harmonic motion,
$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$
Simplifying the second equation of simple harmonic motion asfollows
$x_2 = 5\sqrt{2}\left(\sin 2\pi t \times \frac{1}{\sqrt{2}} + \cos 2\pi t \times \frac{1}{\sqrt{2}}\right)\sqrt{2}$
$= 10\left(\sin 2\pi t \cos\frac{\pi}{4} + \cos 2\pi t \sin\frac{\pi}{4}\right)$
$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$
The ratio of amplitudes of two simple harmonic motions is
$\frac{A_2}{A_1} = \frac{10}{5} = 2$
Thus, amplitude of second motion is 2 times the amplitude in first motion.

Question86

A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?
[1 Sep 2021 Shift 2]

Options:

A. $10 m / s^2$

B. $5 m / s^2$

C. $4 m / s^2$

D. $9.8 m / s^2$

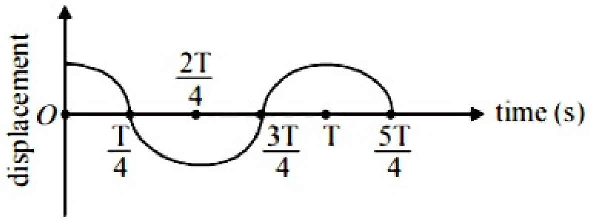
Answer: C

Solution:

Solution:
From the potential energy curve,
$U_{max} = \frac{1}{2}kA^2$
$10 = \frac{1}{2}k(2)^2$
$\Rightarrow k = 5\ N/m$
The length of the simple pendulum, L = 4 m
Time period of spring,
$T = 2\pi\sqrt{\frac{k}{m}}$
Time period of simple pendulum,
$T = 2\pi\sqrt{\frac{I}{g}}$
The time period of simple pendulum is same as the time period ofthe spring oscillation.
$\Rightarrow 2\pi\sqrt{\frac{I}{g}} = 2\pi\sqrt{\frac{k}{m}}$
Substituting the values in the above equations, we get
$2\pi\sqrt{\frac{4}{g}} = 2\pi\sqrt{\frac{5}{5}}$
$g = 4m/s^2$
∴ The acceleration due to the gravity on the planet is $4m/s^2$.

Question87

The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion?
(1) The force is zero at t = $\frac{3T}{4}$
(2) The acceleration is maximum at t = T
(3) The speed is maximum at t = $\frac{T}{4}$
(4) The P.E. is equal to K.E. of the oscillation at t = $\frac{T}{2}$
[Sep. 02, 2020 (II)]

Options:

A. (1),(2) and (4)

B. (2),(3) and (4)

C. (1),(2) and (3)

D. (1) and (4)

Answer: C

Solution:

Solution:
From graph equation of SHM
$X = A\cos\omega t$

(1) At $\frac{3T}{4}$ particle is at mean position.
∴ Acceleration $= 0$, Force $= 0$
(2) At T particle again at extreme position so acceleration ismaximum.
(3) At $t = \frac{T}{4}$, particle is at mean position so velocity is maximum.
Acceleration $= 0$
(4) When $KE = PE$
$\Rightarrow \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$
Here, $A =$ amplitude of SHM
$x =$ displacement from mean position
$\Rightarrow A^2 = 2x^2 \Rightarrow x = \frac{+A}{\sqrt{2}}$
$\Rightarrow \frac{A}{\sqrt{2}} = A\cos\omega t \Rightarrow t = \frac{T}{2}$
$\therefore x = -A$ which is not possible
$\therefore 1$, 2 and 3 are correct.

Question88

An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:
[Sep. 06, 2020 (I)]

Options:

A. $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

B. $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

C. $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$

D. $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

Answer: B

Solution:

Solution:
(b) An elastic wire can be treated as a spring and its spring constant.
$k = \frac{YA}{L}\left[\because Y = \frac{F}{A} / \frac{\Delta l}{l_0}\right]$
Frequency of oscillation,
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

Question89

When a particle of mass m is attached to a vertical spring of spring

constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where ′y′ is measured from the lower end of unstretched spring. Then ω is:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

B. $\sqrt{\frac{g}{y_0}}$

C. $\sqrt{\frac{g}{2y_0}}$

D. $\sqrt{\frac{2g}{y_0}}$

Answer: C

Solution:

Solution:

$y = y_0 \sin^2 \omega t$

$\Rightarrow y = \frac{y_0}{2}(1 - \cos 2\omega t) \quad \left(\because \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}\right)$

$\Rightarrow y - \frac{y_0}{2} = \frac{-y_0}{2}\cos 2\omega t$

$\Rightarrow y = A\cos 2\omega t$

$\therefore$ Amplitude $= \frac{y_0}{2}$

Angular velocity $= 2\omega$

For equilibrium of mass, $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$

Question90

A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become f A. The value of f is :
[Sep. 03, 2020 (II)]

Options:

A. $\frac{1}{\sqrt{2}}$

B. 1

C. $\frac{1}{2}$

D. $\sqrt{2}$

Answer: A

Solution:

Solution:

Potential energy of spring $= \frac{1}{2}kx^2$

Here, $x =$ distance of block from mean position,
$k =$ spring constant

At mean position, potential energy $= \frac{1}{2}kA^2$

At equilibrium position, half of the mass of block breaks off, so its potential energy becomes half.

Remaining energy $= \frac{1}{2}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}kA^2$

Here, $A' =$ New distance of block from mean position

$\Rightarrow A' = \frac{A}{\sqrt{2}}$

Question91

The position co-ordinates of a particle moving in a 3 − D coordinate system is given by
$x = a\cos\omega t$
$y = a\sin\omega t$
and $z = a\omega t$
The speed of the particle is:
[9 Jan 2019, II]

Options:

A. $\sqrt{2}a\omega$

B. $a\omega$

C. $\sqrt{3}a\omega$

D. $2a\omega$

Answer: A

Solution:

Solution:
Here, $v_x = -a\omega\sin\omega t$, $v_y = a\omega\cos\omega t$ and $v_z = a\omega$

$\Rightarrow v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$

$\Rightarrow v = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$

$v = \sqrt{2}a\omega$

Question92

A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210s will be
[11 Jan 2019, I]

Options:

A. $\frac{1}{9}$

B. 1

C. 2

D. $\frac{1}{3}$

Answer: D

Solution:

Solution:

Kinetic energy, $k = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$

Potential energy, $U = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$

$\frac{k}{U} = \cot^2 \omega t = \cot^2 \frac{\pi}{90}(210) = \frac{1}{3}$

Question93

A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2.
[11 Jan 2019, II]

Options:

A. $K_2 = 2K_1$

B. $K_2 = \frac{K_1}{2}$

C. $K_2 = \frac{K_1}{4}$

D. $K_2 = K_1$

Answer: A

Solution:

Solution:

$K = \frac{1}{2}m\omega^2x^2$

$\Rightarrow K_{max} = \frac{1}{2}m\omega^2A^2$

$A = L\theta$

$\omega = \sqrt{\frac{g}{L}}$

$\Rightarrow K = \frac{1}{2}m \cdot \frac{g}{L} \cdot L^2\theta^2$

$= \frac{1}{2}mgL\theta^2$

$\therefore \frac{K_1}{K_2} = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$

$\therefore K_1K_2 = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$

Question94

A particle is executing simple harmonic motion (SHM) of amplitude A, along the x -axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be:
[9 Jan 2019, II]

Options:

A. $\frac{A}{2}$

B. $\frac{A}{2\sqrt{2}}$

C. $\frac{A}{\sqrt{2}}$

D. A

Answer: C

Solution:

Solution:

Potential energy $(U) = \frac{1}{2}kx^2$

Kinetic energy $(K) = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, $U = k$

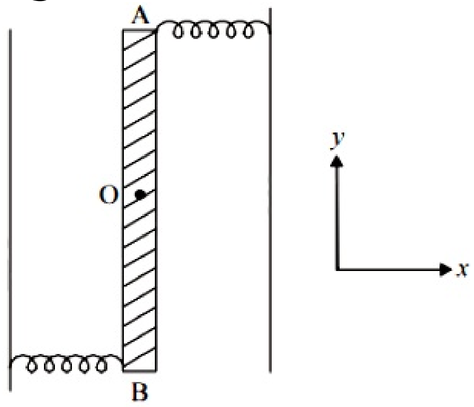
$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\Rightarrow x^2 = A^2$ or, $x = \pm\frac{A}{\sqrt{2}}$

Question95

Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre ' O ' and can rotate frreely

in horizontal plane. The other ends of two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:



[12 Jan 2019, I]

Options:

A. $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

B. $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

C. $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

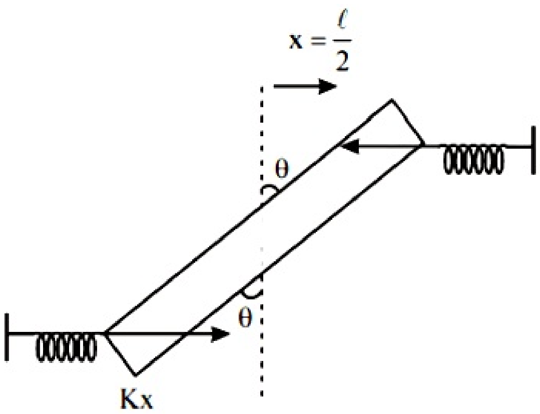
D. $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Answer: C

Solution:

Solution:
Net torque due to spring force:

$\tau = -2Kx\frac{l}{2}\cos\theta$



$\Rightarrow \tau = \left(\frac{Kl^2}{2}\right)\theta = -C\theta$ $\left[\because \text{let } C = \frac{Kl^2}{2}\right]$

⇒ So, frequency of resulting oscillations

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{Kl^2}{2}}{\frac{Ml^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

Question96

A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. The M is given by:
[12 Jan 2019, I]

Options:

A. $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

B. $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

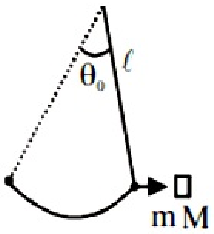
C. $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

D. $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

Answer: B

Solution:

Solution:



Velocity before colision
$v=\sqrt{2gl(1-\cos\theta_0)}$
Velocity after colision
$v_1=\sqrt{2gl(1-\cos\theta_1)}$
Using momentum conservation
$mv=MV_m-mV_1$
$m\sqrt{2gl(1-\cos\theta_0)}=MV_m-m\sqrt{2gl(1-\cos\theta)}$
$\Rightarrow m\sqrt{2gl}\{\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\}=MV_m$
and $e=1=\frac{V_m+\sqrt{2gl(1-\cos\theta_1)}}{\sqrt{2gl(1-\cos\theta_0)}}$
$\sqrt{2gl}(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1})=V_m$ ......(i)
$m\sqrt{2gl}(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1})=MV_M$ ......(ii)
Dividing (ii) by (i) we get
$\frac{(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1})}{(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1})}=\frac{M}{m}$
By componendo and dividendo rule

$$\frac{m-M}{m+M}=\frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}}=\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow\frac{M}{m}=\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\Rightarrow M=m\frac{\theta_0-\theta_1}{\theta_0+\theta_1}$$

Question97

A simple harmonic motion is represented by
$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$cm
The amplitude and time period of the motion are :
[12 Jan 2019, II]

Options:

A. 10cm, $\frac{2}{3}$s

B. 10cm, $\frac{3}{2}$ s

C. 5cm, $\frac{3}{2}$s

D. 5cm, $\frac{2}{3}$s

Answer: A

Solution:

Solution:
Given: $y = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]$
$\Rightarrow y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$
$\therefore$ Amplitude $= 10$cm
Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3}$s

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Question98

A simple pendulum of length 1m is oscillating with an angular frequency 10 rad / s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1rad / s and an amplitude of 10^{-2}m. The relative change in the angular frequency of the pendulum is best given by:
[11 Jan 2019, II]

Options:

A. 10^{-3} rad / s

B. 1 rad / s

C. 10^{-1} rad / s

D. 10^{-5} rad / s

Answer: A

Solution:

Solution:

Angular frequency of pendulum $\omega = \sqrt{\frac{g}{l}}$

$\therefore$ relative change in angular frequency

$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g}{g}$ [as length remains constant]

$\Delta g = 2A\omega_s^2$[$\omega_s =$ angular frequency of support and, A = amplitude]

$\frac{\Delta\omega}{\omega} = \frac{1}{2} \times \frac{2A\omega_S^2}{g}$

$\Delta\omega = \frac{1}{2} \times \frac{2 \times 1^2 \times 10^{-2}}{10} = 10^{-3}$ rad / sec

----

Question99

The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be:
[11 Jan 2019, II]

Options:

A. $\frac{\sqrt{3}}{2}$s

B. $\frac{2}{\sqrt{3}}$s

C. $\frac{3}{2}$s

D. $2\sqrt{3}$s

Answer: D

Solution:

Solution:

Acceleration due to gravity $g = \frac{GM}{R^2}$

$\frac{g_p}{g_e} = M_p M_e \left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$

$\Rightarrow T_p = 2\sqrt{3}$s

Question100

A particle executes simple harmonic motion with an amplitude of 5cm. When the particle is at 4cm from the mean position, the magnitude of

its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is:
[10 Jan 2019, II]

Options:

A. $\frac{4\pi}{3}$

B. $\frac{3}{8}\pi$

C. $\frac{8\pi}{3}$

D. $\frac{7}{3}\pi$

Answer: C

Solution:

Solution:
Velocity, $v = \omega\sqrt{A^2 - x^2}$ ....(i)
acceleration, $a = -\omega^2 x$ ....(ii)
and according to question,
$|v| = |a|$
$\Rightarrow \omega\sqrt{A^2 - x^2} = \omega^2 x$
$\Rightarrow A^2 - x^2 = \omega^2 x^2$
$\Rightarrow 5^2 - 4^2 = \omega^2(4^2)$
$3 = \omega \times 4 \Rightarrow \omega = \frac{3}{4}$
$\therefore T = 2\pi / \omega = \frac{2\pi}{3/4} = \frac{8\pi}{3}$

Question101

A cylindrical plastic bottle of negligible mass is filled with 310ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5cm then omega is close to: (density of water $= 10^3 kg/m^3$)
[10 Jan 2019, II]

Options:

A. $3.75 rad\, s^{-1}$

B. $1.25 rad\, s^{-1}$

C. $2.50 rad\, s^{-1}$

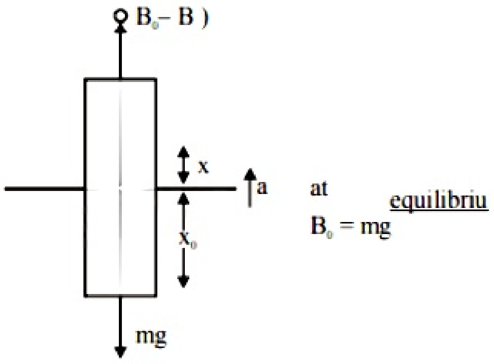
D. $5.00 rad\, s^{-1}$

E. (Bonus)

Answer: E

Solution:

Solution:



Extra boyant force $= \rho Axg$

$B_0 + B = mg + ma$

$\therefore B = ma = \rho Axg = (\pi r^2 \rho g)x$

$a = \frac{(\pi r^2 \rho g)x}{m}$

using, $a = \omega^2 x$

$\Rightarrow \omega = \sqrt{\frac{\omega r^2 \rho g}{m}}$

$W \simeq 7.95 \text{rad s}^{-1}$

Question102

A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m / M is close to :
[9 Jan 2019, II]

Options:

A. 0.77

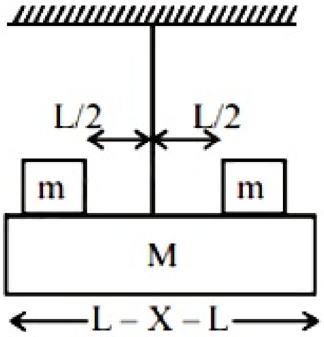
B. 0.57

C. 0.37

D. 0.17

Answer: C

Solution:



$f_1 = \frac{1}{2\pi}\sqrt{\frac{C}{I}} \ldots\ldots (i)$

$= \frac{1}{2}\sqrt{3CML^2}$

$f_2 = \frac{1}{2\pi}\sqrt{CL^2\left(\frac{M}{3} + \frac{M}{2}\right)} \ldots\ldots (ii)$

As frequency reduces by 80%

$\therefore f_2 = 0.8 f_I \Rightarrow \frac{f_2}{f_1} = 0.8 \ldots\ldots. (iii)$

Solving equations (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

As frequency reduces by 80%

$\therefore f_2 = 0.8 f_I \Rightarrow \frac{f_2}{f_1} = 0.8 \ldots\ldots. (iii)$

Solving equations (i),(ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

Question103

The displacement of a damped harmonic oscillator is given by $x(t) = e^{0.1t} . \cos(10\pi t + \varphi)$. Here tis in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to :
[9 Jan 2019, II]

Options:

A. 4s

B. 7s

C. 13s

D. 27s

Answer: B

Solution:

Solution:

Amplitude of vibration at time t = 0 is given by

$A = A_0 e^{-0.1 \times 0} = 1 \times A_0 = A$

also at t = t, if $A = \frac{A_0}{2}$

$\Rightarrow \frac{1}{2} = e^{-0.1t}$

$t = 10 \ln 2 \simeq 7s$

Question104

A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his

centre of mass moves by a distance l (l < < L), is close to :
[12 April 2019, II]

Options:

A. mgl $(1 - \theta_0{}^2)$

B. mgl $(1 + \theta_0{}^2$ rht)

C. mgl

D. M gl $\left(1 + \frac{\theta_0{}^2}{2}\right)$

Answer: B

Solution:

Solution:

Question105

A simple pendulum oscillating in air has period T . The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :
[9 April 2019 I]

Options:

A. $2T\sqrt{\frac{1}{10}}$

B. $2T\sqrt{\frac{1}{14}}$

C. $4T\sqrt{\frac{1}{15}}$

D. $4T\sqrt{\frac{1}{14}}$

Answer: C

Solution:

Solution:
$T = 2\pi\sqrt{\frac{l}{g}}$
When immersed non viscous liquid
$a_{mt} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$

Now $T' = 2\pi\sqrt{\frac{l}{0_{net}}} = 2\pi\sqrt{\frac{l}{\sqrt{\frac{15g}{16}}}} = \frac{4}{\sqrt{15}}T$

Question106

A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to: [8 April 2019, II]

Options:

A. 50s

B. 100s

C. 20s

D. 10s

Answer: C

Solution:

Solution:
Time of half the amplitude is = 2s
Using, $A = A_0 e^{-kt}$
$\frac{A_0}{2} = A_e e^{-k \times 2}$ ......(i)
and $\frac{A_0}{1000} = A_e e^{-kt}$ ......(ii)
Dividing (i) by (ii) and solving, we get
t = 20s

Question107

Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.
$x(t) = A\sin(at + \delta)$
$y(t) = B\sin(bt)$
Identify the correct match below
[Online April 15, 2018]

Options:

A. Parameters: $A = B$, $a = 2b$; $\delta = \frac{\pi}{2}$; Curve: Circle

B. Parameters: $A = B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Line

C. Parameters: $A \neq B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Ellipse

D. Parameters: $A \neq B$, $a = b$; $\delta = 0$; Curve: Parabola

Answer: C

Solution:

Solution:
From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A\sin(at + \delta)$
$y = B\sin(bt + r)$
Clearly $A \neq B$ hence ellipse.

Question108

A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} / sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver $= 108$ and Avagadro number $= 6.02 \times 10^{23}$ gm mol e^{-1}) [2018]

Options:

A. $6.4 N/m$

B. $7.1 N/m$

C. $2.2 N/m$

D. $5.5 N/m$

Answer: B

Solution:

Solution:
As we know, frequency in SHM $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$
where $m =$ mass of one atom
Mass of one atom of silver, $= \frac{108}{(6.02 \times 10^{23})} \times 10^{-3}$ kg

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

Solving we get, spring constant,
$K = 7.1 N/m$

Question109

A particle executes simple harmonic motion and is located at $x = a$, b and c at $\times t_0$, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is [Online April 16, 2018]

Options:

A. $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$

B. $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{3c}\right)$

C. $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$

D. $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

Answer: D

Solution:

Solution:
Using $y = A\sin\omega t$
$a = A\sin\omega t_0$
$b = A\sin 2\omega t_0$
$c = A\sin 3\omega t_0$
$a + c = A[\sin\omega t_0 + A\sin 3\omega t_0] = 2A\sin 2\omega t_0 \cos\omega t_0$
$\frac{a+c}{b} = 2\cos\omega t_0$
$\Rightarrow \omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

Question110

An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x-X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is:
(M = 10, m = 5, u = 1, k = 1),
[Online April 16, 2018]

Options:

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{2}{3}$

D. $\sqrt{\frac{3}{5}}$

Answer: B

Solution:

Solution:
In first collision mu momentum will be imparted to system, in second collision when momentum of $(M + m)$ is in opposite direction mu momentum of particle will make its momentum zero.
On 13^{th} collision, [m]→[M+12]; [M + 13m]→V

$mu = (M + 13m)v \Rightarrow v = \frac{mu}{M + 13m} = \frac{u}{15}$

$v = \omega A \Rightarrow \frac{u}{15} = \sqrt{\frac{K}{M - 13m}} \times A$

Putting value of M, m, u and K we get amplitude $A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$

Question111

The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $10s^{-1}$. At, $t = 0$ the displacement is 5m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$
[Online April 8,2017]

Options:

A. $500\ m/s^2$

B. $500\sqrt{2}\ m/s^2$

C. $750\ m/s^2$

D. $750\sqrt{2}\ m/s^2$

Answer: B

Solution:

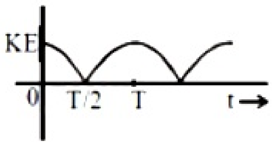
Solution:
Maximum velocity in SHM, $v_{max} = a\omega$
Maximum acceleration in SHM, $A_{max} = a\omega^2$
where a and ω are maximum amplitude and angular frequency.
Given that, $\frac{A_{max}}{v_{max}} = 10$
i.e., $\omega = 10s^{-1}$
Displacement is given by
$x = a\sin(\omega t + \pi/4)$
at $t = 0$, $x = 5$
$5 = a\sin\pi/4$
$5 = a\sin 45° \Rightarrow a = 5\sqrt{2}$
Maximum acceleration $A_{max} = a\omega^2 = 500\sqrt{2}m/s^2$

Question112

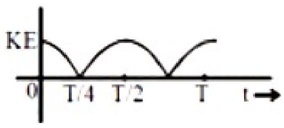
A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like:
[2017]

Options:

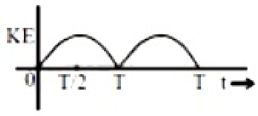
A.



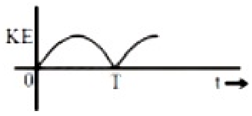
B.



C.



D.



Answer: B

Solution:

Solution:
For a particle executing SHM
At mean position; $t = 0, \omega t = 0, y = 0, V = V_{max} = a\omega$

$\therefore K.E. = KE_{max} = \frac{1}{2}m\omega^2a^2$

At extreme position: $t = \frac{T}{4}, \omega t = \frac{\pi}{2}, y = A, V = V_{min} = 0$

$\therefore K.E. = KE_{min} = 0$

Kinetic energy in SHM, $KE = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2}m\omega^2a^2\cos^2\omega t$

Hence graph (b) correctly depicts kinetic energy time graph.

Question113

A block of mass 0.1kg is connected to an elastic spring of spring constant $640 N m^{-1}$ and oscillates in a medium of constant $10^{-2} kg s^{-1}$. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to :
[Online April 9,2017]

Options:

A. 2s

B. 3.5s

C. 5s

D. 7s

Answer: B

Solution:

Solution:
Since system dissipates its energy gradually, and hence amplitude will also decreases with time according to
$a = a_0 e^{-bt/m}$......(i)
∵ Energy of vibration drop to half of its initial value
(E_0), as $E \propto a^2 \Rightarrow a \propto \sqrt{E}$
$a = \frac{a_0}{\sqrt{2}} \Rightarrow \frac{bt}{m} = \frac{10^{-2}t}{0.1} = \frac{t}{10}$
From eq ^{n} (i), $\frac{a_0}{\sqrt{2}} = a_0 e^{-t/10}$
$\frac{1}{\sqrt{2}} = e^{-t/10}$ or $\sqrt{2} = e^{\frac{t}{10}}$
$\ln\sqrt{2} = \frac{t}{10}$ ∴ t = 3.5 seconds

Question114

In an experiment to determine the period of a simple pendulum of length 1m, it is attached to different spherical bobs of radii r_1 and r_2. The two spherical bobs have uni- form mass distribution. If the relative difference in the periods, is found to be 5×10^{-4}s, the difference in radii, $|r_1 - r_2|$ is best given by:
[Online April 9,2017]

Options:

A. 1cm

B. 0.1cm

C. 0.5cm

D. 0.01cm

Answer: B

Solution:

Solution:
As we know, Time-period of simple pendulum,
$T \propto \sqrt{l}$
differentiating both side, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$
∵ change in length $\Delta l = r_1 - r_2$

$5 \times 10^{-4} = \frac{1}{2}\frac{r_1 - r_2}{1}$
$r_1 - r_2 = 10 \times 10^{-4}$
$10^{-3}m = 10^{-1}cm = 0.1cm$

Question115

A 1kg block attached to a spring vibrates with a frequency of 1Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8kg block placed on the same table. So, the frequency of vibration of the 8kg block is :
[Online April 8, 2017]

Options:

A. $\frac{1}{4}Hz$

B. $\frac{1}{2\sqrt{2}}Hz$

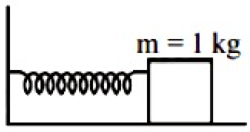
C. $\frac{1}{2}Hz$

D. $2Hz$

Answer: C

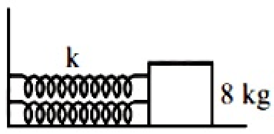
Solution:

Solution:



Frequency of spring (f) $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1Hz$

$\Rightarrow 4\pi^2 = \frac{k}{m}$



If block of mass $m = 1kg$ is attached then, $k = 4\pi^2$
Now, identical springs are attached in parallel with mass $m = 8kg$. Hence,
$k_{eq} = 2k$
$F = \frac{1}{2\pi}\sqrt{\frac{k \times 2}{g}} = \frac{1}{2}Hz$

Question116

A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is:

[2016]

Options:

A. $A\sqrt{3}$

B. $\frac{7A}{3}$

C. $\frac{A}{3}\sqrt{41}$

D. 3A

Answer: B

Solution:

Solution:

We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where $A' =$ final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9} \therefore A' = \frac{7A}{3}$$

Question117

Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time t = 0 one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is:

[Online April 9, 2016]

Options:

A. $\frac{5T}{6}$

B. $\frac{T}{3}$

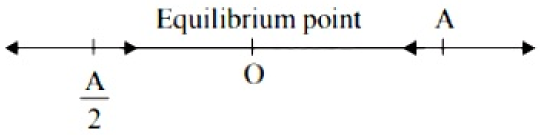
C. $\frac{T}{4}$

D. $\frac{T}{6}$

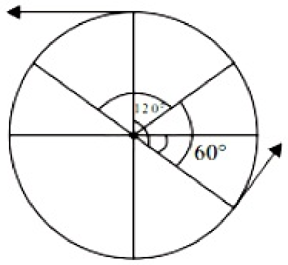
Answer: D

Solution:

Solution:



(at time $t = 0$)



Angle covered to meet $\theta = 60° = \frac{\pi}{3}$ rad

If they cross each other at time t then

$t = \frac{\theta}{2\pi} = \frac{\pi}{3 \times 2\pi}T = \frac{T}{6}$

Question118

A pendulum clock loses 12s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:
[2016]

Options:

A. 30°C; $\alpha = 1.85 \times 10^{-3}$ / °C

B. 55°C; $\alpha = 1.85 \times 10^{-2}$ / °C

C. 25°C; $\alpha = 1.85 \times 10^{-5}$ / °C

D. 60°C; $\alpha = 1.85 \times 10^{-4}$ / °C

Answer: C

Solution:

Solution:

Time lost/gained per day $= \frac{1}{2} \propto \Delta\theta \times 86400$ second

$12 = \frac{1}{2}\alpha(40 - \theta) \times 86400$ ......(i)

$4 = \frac{1}{2}\alpha(\theta - 20) \times 86400$ ....(ii)

On dividing we get, $3 = \frac{40 - \theta}{\theta - 20}$

$3\theta - 60 = 40 - \theta$
$4\theta = 100 \Rightarrow \theta = 25°C$

Question119

In an engine the piston undergoes vertical simple harmonic motion with amplitude 7cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to :
[Online April 10,2016]

Options:

A. $0.7 Hz$

B. $1.9 Hz$

C. $1.2 Hz$

D. $0.1 Hz$

Answer: B

Solution:

Solution:
Washer contact with piston ⇒ $N = 0$ Given Amplitude $A = 7cm = 0.07m$
$a_{max} = g = \omega^2 A$
The frequency of piston
$f = \frac{\omega}{2\pi} = \sqrt{\frac{g}{A}}\frac{1}{2\pi} = \sqrt{\frac{1000}{7}}\frac{1}{2\pi} = 1.9 Hz$

Question120

A simple harmonic oscillator of angular frequency 2rad s^{-1} is acted upon by an external force $F = \sin t$ N. If the oscillator is at rest in its equilibrium position at t = 0, its position at later times is proportional to:
[Online April 10, 2015]

Options:

A. $\sin t + \frac{1}{2}\cos 2t$

B. $\cos t - \frac{1}{2}\sin 2t$

C. $\sin t - \frac{1}{2}\sin 2t$

D. $\sin t + \frac{1}{2}\sin 2t$

Answer: C

Solution:

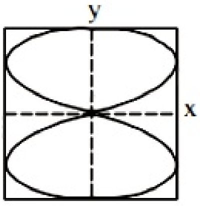
Solution:
As we know,
$F = ma \Rightarrow a \propto F$
or, $a \propto \sin t$
$\Rightarrow \frac{dv}{dt} \propto \sin t$
$\Rightarrow \int_0^0 dV \propto \int_0^t \sin t\, dt$
$V \propto -\cos t + 1$
$\int_0^x dx = \int_0^t (-\cos t + 1) dt$
$x = \sin t - \frac{1}{2}\sin 2t$

Question121

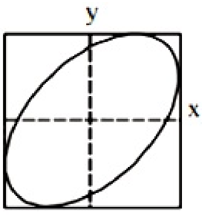
x and y displacements of a particle are given as $x(t) = a \sin \omega t$ and $y(t) = a \sin 2\omega t$. Its trajectory will look like:
[Online April 10, 2015]

Options:

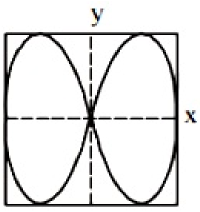
A.



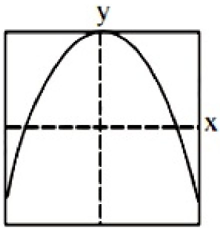
B.



C.



D.



Answer: C

Solution:

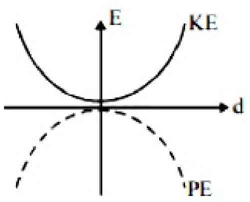
Solution:
At t = 0, x(t) = 0; y(t) = 0
x(t) is a sinusoidal function At
$t = \frac{\pi}{2\omega}$; x(t) = a and y(t) = 0
Hence trajectory of particle will look like as (c).

Question122

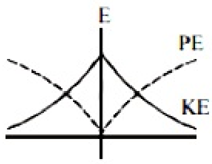
For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) [2015]

Options:

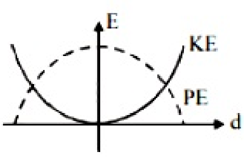
A.



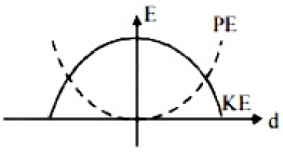
B.



C.



D.



Answer: D

Solution:

Solution:

$K.E = \frac{1}{2}k(A^2 - d^2)$

and $P.E. = \frac{1}{2}kd^2$

At mean position $d = 0$. At extreme positions $d = A$

Question123

A pendulum with time period of 1s is losing energy. At certain time its energy is 45J . If after completing 15 oscillations, its energy has become 15J , its damping constant (in s^{-1}) is :
[Online April 11, 2015]

Options:

A. $\frac{1}{2}$

B. $\frac{1}{30}\ln 3$

C. 2

D. $\frac{1}{15}\ln 3$

Answer: D

Solution:

Solution:

As we know, $E = E_0 e^{-\frac{bt}{m}}$

$15 = 45e^{-\frac{b15}{m}}$

[As no. of oscillations $= 15$ so $t = 15sec$]

$\frac{1}{3} = e^{-\frac{b15}{m}}$

Taking log on both sides

$\frac{b}{m} = \frac{1}{15}\ln 3$

Question124

A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
(g = gravitational acceleration)
[2015]

Options:

A. $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

B. $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

C. $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

D. $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

Answer: C

Solution:

Solution:

As we know, time period, $T = 2\pi\sqrt{\frac{l}{g}}$

When additional mass M is added then

$T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}}$

$\frac{T_M}{T} = \sqrt{\frac{l+\Delta l}{l}}$

$\Rightarrow \left(\frac{T_M}{T}\right)^2 = \frac{l+\Delta l}{l}$

or , $\left(\frac{T_M}{T}\right)^2 = 1+\frac{Mg}{AY}\left[\because \Delta l = \frac{Mgl}{AY}\right]$

$\therefore \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

Question125

A body is in simple harmonic motion with time period half second ($T = 0.5s$) and amplitude one cm($A = 1cm$). Find the average velocity in the interval in which it moves form equilibrium position to half of its amplitude.
[Online April 19,2014]

Options:

A. $4cm / s$

B. $6cm / s$

C. $12cm / s$

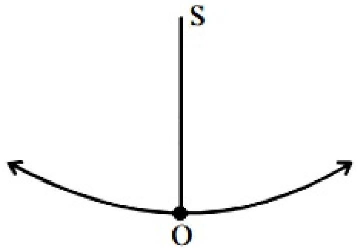
D. $16cm / s$

Answer: C

Solution:

Solution:
Given: Time period, $T = 0.5$ sec

Amplitude, $A = 1\text{cm}$
Average velocity in the interval in which body moves from equilibrium to half of its amplitude, $v = ?$



Time taken to a displacement $A/2$ where A is the amplitude of oscillation from the mean position 'O' is $\frac{T}{12}$

Therefore, time, $t = \frac{0.5}{12}\text{sec}$

Displacement, $s = \frac{A}{2} = \frac{1}{2}\text{cm}$

$\therefore$ Average velocity, $v = \frac{\frac{A}{2}}{t} = \frac{\frac{1}{2}}{\frac{0.5}{12}} = 12\text{cm}/\text{s}$

Question126

Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants?
[Online April 12,2014]

Options:

A. $a + bx - cx^2$

B. bx^2

C. $a - bx + cx^2$

D. $-bx$

Answer: D

Solution:

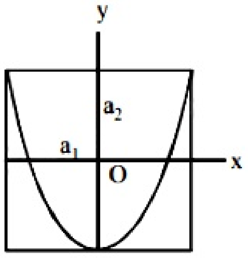
Solution:
In linear S.H.M., the restoring force acting on particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.
i.e., $F \propto x$
or $F = -bx$ where b is a positive constant.

Question127

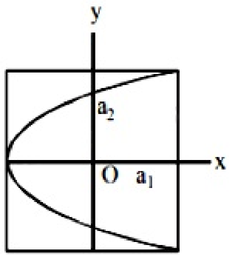
A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by; $x = a_1 \cos\omega t$ and $y = a_2 \cos 2\omega t$ traces a curve given by:
[Online April 9, 2014]

Options:

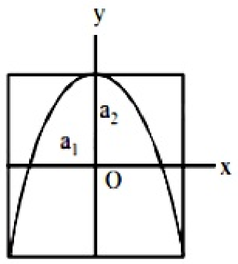
A.



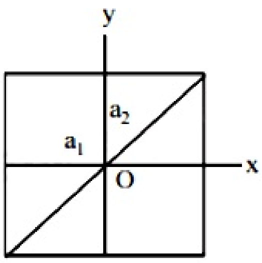
B.



C.



D.



Answer: B

Solution:

Two perpendicular S.H.Ms are

$x = a_1 \cos \omega t$ ....(1)

and $y = a_2 2 \cos \omega t$ ....(2)

From eqn (1)

$\frac{x}{a_1} = \cos wt$

and from eqn (2)

$\frac{y}{a_2} = 2 \cos \omega t$

$\therefore y = 2\frac{a_2}{a_1}x$

Question128

A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then:
[2014]

Options:

A. amplitude of motion is 3a

B. time period of oscillations is 8τ

C. amplitude of motion is 4a

D. time period of oscillations is 6τ

Answer: D

Solution:

Solution:
In simple harmonic motion, starting from rest,
At $t = 0, x = A$
$x = A\cos\omega t$ ....(i)
When $t = \tau, x = A - a$
When $t = 2\tau, x = A - 3a$
From equation (i)
$A - a = A\cos\omega\tau$ ....(ii)
$A - 3a = A\cos 2\omega\tau$ ....(iii)
As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$ ....(iv)
From equation (ii), (iii) and (iv)

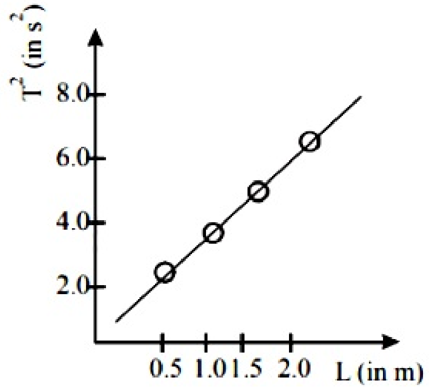
$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$
$\Rightarrow 2a^2 = aA$
$\Rightarrow A = 2a$
$\Rightarrow \frac{a}{A} = \frac{1}{2}$
Now, $A - a = A\cos\omega\tau$
$\Rightarrow \cos\omega\tau = \frac{A-a}{A}$
$\Rightarrow \cos\omega\tau = \frac{1}{2}$ or $\frac{2\pi}{T}\tau = \frac{\pi}{3}$
$\Rightarrow T = 6\tau$

Question129

In an experiment for determining the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure.



What is the value of g at the place?
[Online April 19, 2014]

Options:

A. $9.81 m / s^2$

B. $9.87 m / s^2$

C. $9.91 m / s^2$

D. $10.0 m / s^2$

Answer: B

Solution:

Solution:
From graph it is clear that when
$L = 1m, T^2 = 4s^2$
As we know,

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2}$$

$$= 4 \times \left(\frac{22}{7}\right)^2 \times \frac{1}{4} = \left(\frac{22}{7}\right)^2$$

$$\therefore g = \frac{484}{49} = 9.87 m / s^2$$

Question130

The amplitude of a simple pendulum, oscillating in air with a small spherical bub, decreases from 10cm108cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3 . The time in which amplitude of this pendulum will reduce from 10cm to 5cm in carbon dioxide will be close to ($\ln 5 = 1.601, \ln 2 = 0.693$).
[Online April 9, 2014]

Options:

A. 231s

B. 208s

C. 161s

D. 142s

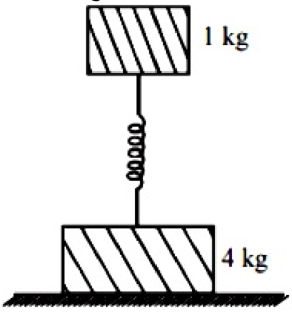
Answer: D

Solution:

Solution:
As we know,
$x = x_0 e^{-bt/2m}$
From question,
$8 = 10e^{-\frac{40b}{2m}}$ ....(i)
Similarly, $5 = 10e^{-\frac{bt}{2m}}$ ....(ii)
Solving eqns (i) and (ii) we get
$t \cong 142s$

Question131

Two bodies of masses 1kg and 4kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25rad / s, and amplitude 1.6cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take $g = 10ms^{-2}$)



[Online April 9, 2014]

Options:

A. 20N

B. 10N

C. 60N

D. 40N

Answer: C

Solution:

Solution:
Mass of bigger body $M = 4kg$
Mass of smaller body $m = 1kg$
Smaller mass ($m = 1kg$)
executes S.H.M of angular frequency $\omega = 25$ rad / s
Amplitude $x = 1.6cm = 1.6 \times 10^{-2}$

As we know,

$T = 2\pi\sqrt{\frac{m}{K}}$

or, $2\pi\omega = 2\pi\sqrt{\frac{m}{K}}$

or, $\frac{1}{25} = \sqrt{\frac{1}{K}}$ $[\because m = 1kg; \omega = 25 rad/s]$

or, $K = 625 N m^{-1}$.

The maximum force exerted by the system on the floor $= Mg + Kx + mg$

$= 4 \times 10 + 625 \times 1.6 \times 10^{-2} + 1 \times 10$

$= 40 + 10 + 10$

$= 60N$

Question132

The angular frequency of the damped oscillator is given by, $\omega = \sqrt{\left(\frac{k}{m} - \frac{r^2}{4m^2}\right)}$ where k is the spring constant, mis the mass of the oscillator and r is the damping constant.
If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period compared to the undamped oscillator is approximately as follows:
[Online April 11, 2014]

Options:

A. increases by 1%

B. increases by 8%

C. decreases by 1%

D. decreases by 8%

Answer: B

Solution:

Solution:
The change in time period compared to the undamped oscillator increases by 8%.

Question133

The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10 s it will decrease to α times its original magnitude, where α equals
[2013]

Options:

A. 0.7

B. 0.81

C. 0.729

D. 0.6

Answer: C

Solution:

Solution:

$\because A = A_0 e^{-\frac{bt}{2m}}$

(where, $A_0 =$ maximum amplitude)

According to the questions, after 5 second,

$0.9 \sim A_0 = A_0 e^{-\frac{b(5)}{2m}}$ ....(i)

After 10 more second,

$A = A_0 e^{-\frac{b(15)}{2m}}$ ....(ii)

From eq $^{\text{mathrm n}}$ s (i) and (ii)

$A = 0.729 A_0$

$\therefore \alpha = 0.729$

Question134

An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency
[2013]

Options:

A. $\frac{1}{2\pi} \frac{A\gamma P_0}{V_0 M}$

B. $\frac{1}{2\pi} \frac{V_0 M P_0}{A^2 \gamma}$

C. $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{M V_0}}$

D. $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A \gamma P_0}}$

Answer: C

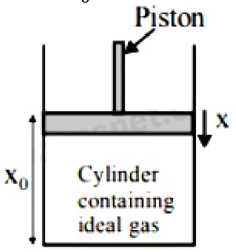
Solution:

Solution:

$\frac{Mg}{A} = P_0$

$P_0V_0^{\gamma} = PV^{\gamma}$
$Mg = P_0A$ .....(i)
Let piston is displaced by distance x
$P_0Ax_0^{\gamma} = PA(x_0 - x)^{\gamma}$

$$P = \frac{P_0x_0^{\gamma}}{(x_0 - x)^{\gamma}}$$



$$Mg - \left(\frac{P_0x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right)A = F_{restoring}$$

$$P_0A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) = F_{restoring}\ [x_0 - x \approx x_0]$$

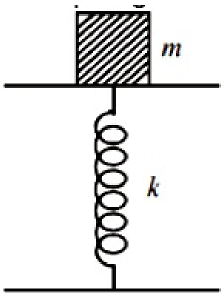
$$F = -\frac{\gamma P_0Ax}{x_0}$$

∴ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A}{x_0M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

Question135

A mass m = 1.0kg is put on a flat pan attached to a vertical spring fixed on the ground. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes simple harmonic motion. The spring constant is 500N / m. What is the amplitude A of the motion, so that the mass m tends to get detached from the pan? (Take $g = 10m/s^2$).
The spring is stiff enough so that it does not get distorted during the motion.



[Online April 22, 2013]

Options:

A. A > 2.0cm

B. A = 2.0cm

C. A < 2.0cm

D. A = 1.5cm

Answer: C

Solution:

Solution:
As $F = -kx$

Question136

Two simple pendulums of length 1m and 4m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to :
[Online April 9,2013]

Options:

A. 2

B. 7

C. 5

D. 3

Answer: A

Solution:

Solution:
Let T_1 and T_2 be the time period of the two pendulums $T_1 = 2\pi\sqrt{\frac{1}{g}}$ and $T_2 = 2\pi\sqrt{\frac{4}{g}}$
As $l_1 < l_2$ therefore $T_1 < T_2$
Let at $t = 0$ they start swinging together. Since their time periods are different, the swinging will not be in unison always.
Only when number of completed oscillations differ by an integer, the two pendulums will again begin to swing together
Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n+1)$ oscillation. For unison swinging
$(n+1)T_1 = nT_2$
$(n+1) \times 2\pi\sqrt{\frac{1}{g}} = (n) \times 2\pi\sqrt{\frac{4}{g}}$
$\Rightarrow n = 1$
$\therefore n + 1 = 1 + 1 = 2$

Question137

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density sigma at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be :
[Online April 25, 2013]

Options:

A. Smaller than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

B. $2\pi\sqrt{\frac{M}{k}}$

C. Larger than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

D. $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

Answer: A

Solution:

Solution:

Question138

Bob of a simple pendulum of length l is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then : [Online April 23, 2013]

Options:

A. $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

B. $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

C. $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

D. $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

Answer: D

Solution:

Solution:
When the pendulum is oscillating over a current carrying coil, and when the direction of oscillating pendulum bob is opposite to the direction of current. Its instantaneous acceleration increases.

Hence time period $T < 2\pi\sqrt{\frac{l}{g}}$

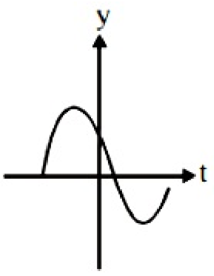
and damping is larger than in air alone due energy dissipation.

Question139

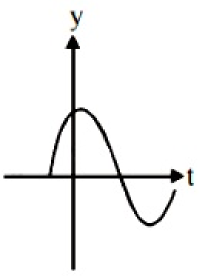
The displacement $y(t) = A\sin(\omega t + \phi)$ of a pendulum for $\phi = \frac{2\pi}{3}$ is correctly represented by
[Online May 19, 2012]

Options:

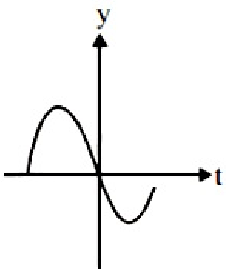
A.



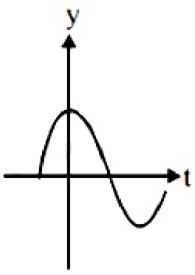
B.



C.



D.



Answer: A

Solution:

Solution:
Displacement $y(t) = A\sin(wt + \phi)$[Given]
For $\phi = \frac{2\pi}{3}$
at $t = 0$; $y = A\sin\phi = A\sin\frac{2\pi}{3}$
$= A\sin 120° = 0.87A$ [$\because \sin 120° \simeq 0.866$]
Graph (a) depicts $y = 0.87A$ at $t = 0$

Question140

This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
If two springs S_1 and S_2 of force constants k_1 and k_2 respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2
Statement 1: If stretched by the same amount work done on S_1
Statement 2: $k_1 < k_2$
[2012]

Options:

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true, Statement 2 is false.

C. Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1

D. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

Answer: B

Solution:

Solution:
Work done, $w = \frac{1}{2}kx^2$
Work done by spring S_1, $w_1 = \frac{1}{2}k_1x^2$
Work done by spring S_2, $w_2 = \frac{1}{2}k_2x^2$
Since $w_1 > w_2$ Thus $(k_1 > k_2)$

Question141

If a simple pendulum has significant amplitude (up to a factor of 1 / e of original) only in the period between t = 0 s to t = τs, then tau may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum in second is (assuming damping is small)
[2012]

Options:

A. $\frac{0.693}{b}$

B. b

C. $\frac{1}{b}$

D. $\frac{2}{b}$

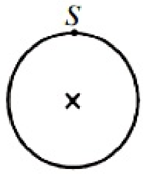
Answer: D

Solution:

The equation of motion for the pendulum, for damped harmonic motion
$F = -kx - bv$
$\Rightarrow ma + kx + bv = 0$
$\Rightarrow m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$
$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = 0$
$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$ ....(i)
Let $x = e^{\lambda t}$ is the solution of the equation (1)
$\frac{dx}{dt} = \lambda e^{\lambda t} \Rightarrow \frac{d^2x}{dt^2} = \lambda^2 e^{\lambda t}$
Substituting in the equation (1)
$\lambda^2 e^{\lambda t} + \frac{b}{m}\lambda e^{\lambda t} + \frac{k}{m}e^{\lambda t} = 0$
$\lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$
$\lambda = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{k}{m}}}{2} = \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$
Solving the equation (1) for x, we have
$x = e^{\frac{-b}{2m}}$
$\omega = \sqrt{\omega_0{}^2 - \lambda^2}$ where $\omega_0 = \frac{k}{m}, \lambda = \frac{+b}{2}$
The average life $= \frac{1}{\lambda} = \frac{2}{b}$

Question142

A ring is suspended from a point S on its rim as shown in the figure. When displaced from equilibrium, it oscillates with time period of 1 second. The radius of the ring is (take $g = \pi^2$)



[Online May 19, 2012]

Options:

A. 0.15m

B. 1.5m

C. 1.0m

D. 0.5m

Answer: A

Solution:

Question143

Two particles are executing simple harmonic motion of the same amplitude A and frequency omega along the x -axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is: [2011]

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: A

Solution:

Let, $x_1 = A \sin \omega t$ and $x_2 = A \sin(\omega t + \phi)$

$x_2 - x_1 = 2A \cos\left(\omega t + \frac{\phi}{2}\right) \sin\frac{\phi}{2}$

The above equation is SHM with amplitude $2A \sin\frac{\phi}{2}$

$\therefore 2A \sin\frac{\phi}{2} = A$

$\Rightarrow \sin\frac{\phi}{2} = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

Question144

A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is:

[2011]

Options:

A. $\frac{M + m}{M}$

B. $\left(\frac{M}{M + m}\right)^{\frac{1}{2}}$

C. $\left(\frac{M + m}{M}\right)^{\frac{1}{2}}$

D. $\frac{M}{M + m}$

Answer: C

Solution:

Solution:
At mean position, $F_{net} = 0$
Therefore, by principal of conseruation of linear momentum.
$\therefore M v_1 = (M + m)v_2$
$M w, a, = (M + m)w_2 a_2$
$M A_1 \sqrt{\frac{k}{M}} = (M + m)A_2 \sqrt{\frac{k}{m + M}}$
$\therefore \left(V = A \sqrt{\frac{k}{M}}\right)$
$\Rightarrow A_1\sqrt{M} = A_2\sqrt{M + m}$
$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{m + M}{M}}$

Question145

A wooden cube (density of wood ' d ') of side ' l ' floats in a liquid of density ' ρ ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'
[2011 RS]

Options:

A. $2\pi \sqrt{\frac{l d}{\rho g}}$

B. $2\pi \sqrt{\frac{l \rho}{d g}}$

C. $2\pi \sqrt{\frac{l d}{(\rho - d)g}}$

D. $2\pi \sqrt{\frac{l \rho}{(\rho - d)g}}$

Answer: A

Solution:

Let the cube be at a depth x from the equilibrium position.
Force acting on the cube = up thrust on the portion of length x.
$F = -\rho l^2 x g$[∴ mass density X volume] .... (i)
Clearly $F \propto -x$, Hence it is a SHM.
Equation of SHM is $F = -kx$ .....(ii)
Comparing equation (i) and (ii) we have
$k = \rho l^2 g$
Now, Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
$\Rightarrow T = 2\pi\sqrt{\frac{l^3 d}{\rho l^2 g}}$
$= 2\pi\sqrt{\frac{ld}{\rho g}}$
Comparing the above equation with
$a = -\omega^2 x$, we get
$\therefore \omega = \sqrt{\frac{\rho g}{d\,l}} \Rightarrow T = 2\pi\sqrt{\frac{ld}{\rho g}}$

Question146

If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T , then, which of the following does not change with time? [2009]

Options:

A. aT / x

B. $aT + 2\pi v$

C. aT / v

D. $a^2T^2 + 4\pi^2 v^2$

Answer: A

Solution:

Solution:
For an SHM, the acceleration
$a = -\omega^2 x$ where ω^2 is a constant.
$a = \frac{-4\pi^2 x}{T^2} \Rightarrow \frac{aT}{x} = \frac{-4\pi^2}{T}$
The time period T is also constant. Therefore, $\frac{aT}{x}$ is a constant.

Question147

A point mass oscillates along the x -axis according to the law $x = x_0 \cos(\omega t - \pi / 4)$. If the acceleration of the particle is written as

$a = A\cos(\omega t + \delta)$, then
[2007]

Options:

A. $A = x_0\omega^2$, $\delta = 3\pi / 4$

B. $A = x_0$, $\delta = -\pi / 4$

C. $A = x_0\omega^2$, $\delta = \pi / 4$

D. $A = x_0\omega^2$, $\delta = -\pi / 4$

Answer: A

Solution:

Solution:
Given,
Displacement, $x = x_0 \cos\left(\omega t - \pi\overline{4}\right)$
$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$
Acceleration,
$a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$
$= x_0\omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$
$= x_0\omega^2 \cos\left(\omega t + \frac{3\pi}{4}\right)$ ....(1)
Acceleration, $a = A\cos(\omega t + \delta)$ ....(2)
Comparing the two equations, we get
$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$

Question148

A particle of mass m executes simple harmonic motion with amplitude a and frequency v. The average kinetic energy during its motion from the position of equilibrium to the end is
[2007]

Options:

A. $2\pi^2 ma^2 v^2$

B. $\pi^2 ma^2 v^2$

C. $\frac{1}{4} ma^2 v^2$

D. $4\pi^2 ma^2 v^2$

Answer: B

Solution:

The kinetic energy of a particle executing S.H.M. at any instant t is given by

$K = \frac{1}{2}ma^2\omega^2\sin^2\omega t$

where, $m =$ mass of particle
$a =$ amplitude
$\omega =$ angular frequency
$t =$ time

The average value of $\sin^2\omega$ t over a cycle is $\frac{1}{2}$.

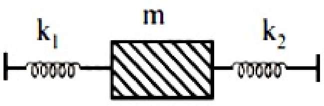
$\therefore K E = \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right)\left(\because <\sin^2\theta> = \frac{1}{2}\right)$

$= \frac{1}{4}m\omega^2a^2 = \frac{1}{4}ma^2(2\pi v)^2(\because \omega = 2\pi v)$

Or $<K> = \pi^2ma^2v^2$

Question149

Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f . If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



[2007]

Options:

A. 2f

B. f / 2

C. f / 4

D. 4f

Answer: A

Solution:

Solution:
(a) The two springs are in parallel.
∴ Effective spring constant,
$k = k_1 + k_2$
Initial frequency of oscillation is given by

$v = \frac{1}{2p}\sqrt{\frac{k_1 + k_2}{m}}$ ......(i)

When both k_1 and k_2 are made four times their original values, the new frequency is given by

$v' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4(k_1 + 4k_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}\right) = 2v$

Question150

The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is
[2007]

Options:

A. 0.25s

B. 0.5s

C. 0.75s

D. 0.125s

Answer: B

Solution:

Solution:
Here, Displacement, $x = 2 \times 10^{-2} \cos \pi t$
Velocity is given by
$v = \frac{dx}{dt} = 2 \times 10^{-2} \pi \sin \pi t$
For the first time, the when velocity becomes maximum,
$\sin \pi t = 1$
$\Rightarrow \sin \pi t = \sin \frac{\pi}{2}$
$\Rightarrow \pi t = \frac{\pi}{2}$ or, $t = \frac{1}{2} = 0.5$ sec

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Question151

A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
[2006]

Options:

A. at the mean position of the platform

B. for an amplitude of $\frac{g}{m^2}$

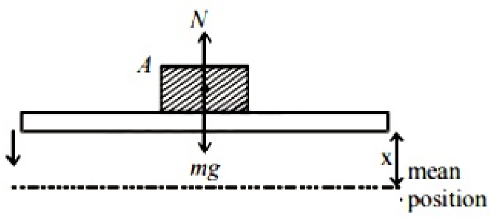
C. for an amplitude of $\frac{g^2}{m^2}$

D. at the highest position of the platform

Answer: B

Solution:

Solution:
For block A to move in SH M .



$mg - N = m\omega^2 x$
where x is the distance from mean position For block to leave contact $N = 0$
$\Rightarrow mg = m\omega^2 x \Rightarrow x = \frac{g}{\omega^2}$

Question152

The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7mm, is 4.4m / s. The period of oscillation is [2006]

Options:

A. 0.01s

B. 10s

C. 0.1s

D. 100s

Answer: A

Solution:

Solution:
Maximum velocity,
$v_{max} = a\omega$
Here, a = amplitude of SHM
ω = angular velocity of SHM
$v_{max} = a \times \frac{2\pi}{T} \therefore \left(\because \omega = \frac{2\pi}{T}\right)$
$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01s$

Question153

**Starting from the origin a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy?
[2006]**

Options:

A. $\frac{1}{6}$s

B. $\frac{1}{4}$s

C. $\frac{1}{3}$s

D. $\frac{1}{12}$s

Answer: A

Solution:

Solution:
K.E. of a body undergoing SHM is given by,
$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t$
Here, a = amplitude of SHM
ω = angular velocity of SHM
Total energy in S.H.M $= \frac{1}{2}ma^2\omega^2$
Given K.E = 75%T.E
$\frac{1}{2}ma^2\omega^2\cos^2\omega t = \frac{75}{100} \times \frac{1}{2}ma^2\omega^2$
$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$
$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6}s$

Question154

The function $\sin^2(\omega t)$ represents [2005]

Options:

A. a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$

B. a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$

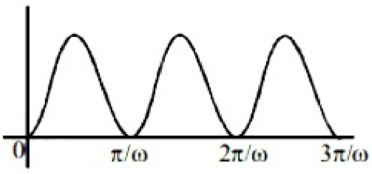
C. a simple harmonic motion with a period $\frac{\pi}{\omega}$

D. a simple harmonic motion with a period $\frac{2\pi}{\omega}$

Answer: A

Solution:

Solution:
Clearly $\sin 2\omega t$ is a periodic function with period $\frac{\pi}{\omega}$



For SHM $\frac{d^2y}{dt^2} \propto -y$
$y = \sin^2\omega t = \frac{1 - \cos 2\omega t}{2}$

$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

$v = \frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t = 2\omega \sin \omega t \cos \omega t$

$= \omega \sin 2\omega t$

Acceleration, $a = \frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$ which is not proportional to $-y$. Hence, it is not in SHM.

Question155

Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$ The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is [2005]

Options:

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{6}$

C. $\frac{\pi}{6}$

D. $\frac{-\pi}{3}$

Answer: B

Solution:

Solution:

Velocity of particle 1 ,

$v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$

Velocity of particle 2

$v_2 = \frac{dy_2}{dt} = -0.1\pi \sin \pi t = 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$

∴ Phase difference of velocity of particle 1 with respect to the velocity of particle 2 is

$= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$

Question156

The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would [2005]

Options:

A. first decrease and then increase to the original value

B. first increase and then decrease to the original value

C. increase towards a saturation value

D. remain unchanged

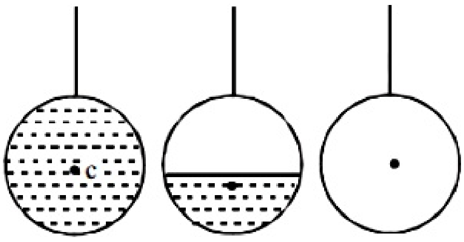
Answer: B

Solution:

Solution:
When plugged hole near the bottom of the oscillating bob gets suddenly unplugged, centre of mass of combination of liquid and hollow portion (at position l), first goes down (to $l+\Delta l$) and when total water is drained out, centre of mass regain its original position (to l),

Time period, $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore$ 'T' first increases and then decreases to original value.



Question157

If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is [2005]

Options:

A. $\frac{2\pi}{\sqrt{\alpha}}$

B. $\frac{2\pi}{\alpha}$

C. $2\pi\sqrt{\alpha}$

D. $2\pi\alpha$

Answer: A

Solution:

Solution:
Standard dlfferentlal equatlon of SHM Is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0 \therefore \omega^2 = \alpha$

or $\omega = \sqrt{\alpha}$

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$

Question158

The total energy of a particle, executing simple harmonic motion is where x is the displacement from the mean position, hence total energy is independent of x
[2004]

Options:

A. independent of x

B. $\propto x^2$

C. $\propto x$

D. $\propto x^{1/2}$

Answer: A

Solution:

Solution:
At any instant the total energy in SHM is $\frac{1}{2}kA_0{}^2 =$ constant,
where $A_0 =$ amplitude
$k =$ spring constant
hence total energy is independent of x.

Question159

The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $\left(\frac{4}{3}\right) \times 1000 kg / m^3$. Which relationship between t and t_0 is true?
[2004]

Options:

A. $t = 2t_0$

B. $t = t_0 / 2$

C. $t = t_0$

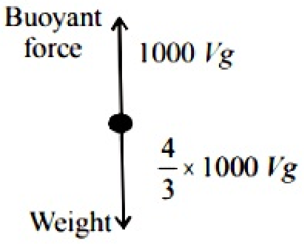
D. $t = 4t_0$

Answer: A

Solution:

Time period, $t = 2\pi\sqrt{\frac{l}{g_{eff}}}$;

In air, $t_0 = 2\pi\sqrt{\frac{l}{g}}$



Net force $= \left(\frac{4}{3} - 1\right) \times 1000V\,g = \frac{1000}{3}V\,g$

$g_{eff} = \frac{1000V\,g}{3 \times \frac{4}{3} \times 1000V} = \frac{g}{4}$

$\therefore t = 2\pi\sqrt{lg / 4} = 2 \times 2\pi\frac{l}{g}$

$t = 2t_0$

Question160

A particle at the end of a spring executes S.H.M with a period t_1. while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T then
[2004]

Options:

A. $T^{-1} = t_1^{-1} + t_2^{-1}$

B. $T^2 = t_1^2 + t_2^2$

C. $T = t_1 + t_2$

D. $T^{-2} = t_1^{-2} + t_2^{-2}$

Answer: B

Solution:

Solution:

Time period for first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$,

Time period for second spring, $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

Force constant of the series combination $k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$

$\therefore$ Time period of oscillation for series combination $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$

$\therefore T = 2\pi\sqrt{\frac{m}{k_2} + \frac{m}{k_1}} = 2\pi\sqrt{\frac{t_2^2}{(2\pi)^2} + \frac{t_1^2}{(2\pi)^2}}$

$\Rightarrow T^2 = t_1^2 + t_2^2$

where x is the displacement from the mean position

Question161

In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force while the energy is maximum for a frequency ω_2 of the force; then
[2004]

Options:

A. $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

B. $\omega_1 > \omega_2$

C. $\omega_1 = \omega_2$

D. $\omega_1 < \omega_2$

Answer: C

Solution:

Solution:
As energy $\propto$ (Amplitude) ^{2}, the maximum for both of them occurs at the same frequency and this is only possible in case of resonance.
In resonance state $\omega_1 = \omega_2$

Question162

A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F (t) proportional to $\cos \omega t (\omega \neq \omega_0)$ is applied to the oscillator. The displacement of the oscillator will be proportional to
[2004]

Options:

A. $\frac{1}{m(\omega_0^2 + \omega^2)}$

B. $\frac{1}{m(\omega_0^2 - \omega^2)}$

C. $\frac{m}{\omega_0^2 - \omega^2}$

D. $\frac{m}{(\omega_0^2 + \omega^2)}$

Answer: B

Solution:

Equation of displacement in forced oscillation is given by

$$y = \frac{F_0}{m(\omega_0^2 - \omega^2)^2}$$

$$= \frac{F_0}{m(\omega_0^2 - \omega^2)}$$

Here damping effect is considered to be zero

$$\therefore x \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

Question163

Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is [2003]

Options:

A. $\sqrt{\frac{k_1}{k_2}}$

B. $\frac{k_2}{k_1}$

C. $\sqrt{\frac{k_2}{k_1}}$

D. $\frac{k_1}{k_2}$

Answer: C

Solution:

Solution:

Maximum velocity during SHM, $V_{max} = A\omega$ But $k = m\omega^2$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

$\therefore$ Maximum velocity of the body in SHM $= A\sqrt{\frac{k}{m}}$

As maximum velocities are equal

$$\therefore A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\Rightarrow A_1\sqrt{k_1} = A_2\sqrt{k_2} \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

Question164

The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is

[2003]

Options:

A. -4

B. 4

C. $4\sqrt{2}$

D. 8

Answer: C

Solution:

Solution:
Displacement, $x = 4(\cos\pi t + \sin\pi t)$
$= \sqrt{2} \times 4\left(\frac{\sin\pi t}{\sqrt{2}} + \frac{\cos\pi t}{\sqrt{2}}\right)$
$= 4\sqrt{2}(\sin\pi t\cos 45° + \cos\pi t\sin 45°)$
$x = 4\sqrt{2}\sin(\pi t + 45°)$
On comparing it with standard equation $x = A\sin(\omega t + \phi)$
we get $A = 4\sqrt{2}$

Question165

A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true ? [2003]

Options:

A. K.E. is maximum when x = 0

B. T . E is zero when x = 0

C. K.E is maximum when x is maximum

D. P.E is maximum when x = 0

Answer: A

Solution:

Solution:
K.E. of simple harmonic motion $= \frac{1}{2}m\omega^2(a^2 - x^2)$

Question166

A mass M is suspended from a spring of negligible mass. The spring is

pulled a little and then released so that the mass executes SHM of time period T . If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is
[2003]

Options:

A. $\frac{3}{5}$

B. $\frac{25}{9}$

C. $\frac{16}{9}$

D. $\frac{5}{3}$

Answer: C

Solution:

Solution:
With mass M , the time period of the spring.

$T = 2\pi\sqrt{\frac{M}{k}}$

With mass $M + m$, the time period becomes,

$T' = 2\pi\sqrt{\frac{M + m}{k}} = \frac{5T}{3}$

$\therefore 2\pi\sqrt{\frac{M + m}{k}} = \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}}$

$\Rightarrow M + m = \frac{25}{9} \times M$

$\Rightarrow 1 + \frac{m}{M} = \frac{25}{9}$

$\Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$

Question167

The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
[2003]

Options:

A. 11%

B. 21%

C. 42%

D. 10%

Answer: D

Solution:

Time period, $T = 2\pi\sqrt{\frac{l}{g}}$

New length, $l' = l + 21\%$ of l

$l' = l + 0.21l$

$\Rightarrow l' = 1.21l$

$T' = 2\pi\sqrt{\frac{1.21l}{g}}$

% increase in length $= \frac{T' - T}{T} \times 100$

$= \frac{\sqrt{1.21l} - \sqrt{l}}{\sqrt{l}} \times 100 = (\sqrt{1.21} - \sqrt{1}) \times 100$

$= (1.1 - 1) \times 100 = 10\%$

Question168

In a simple harmonic oscillator, at the mean position [2002]

Options:

A. kinetic energyisminimum, potential energy is maximum

B. both kinetic and potential energies are maximum

C. kinetic energyismaximum, potential energy is minimum

D. both kinetic and potential energies are minimum

Answer: C

Solution:

Solution:

The kinetic energy (K . E.) of particle in SHM is given by,

$K.E = \frac{1}{2}k(A^2 - x^2)$

Potential energy of particle in SHM is $U = \frac{1}{2}kx^2$

Where $A =$ amplitude and $k = m\omega^2$

$x =$ displacement from the mean position

At the mean position $x = 0$

$\therefore K.E. = \frac{1}{2}kA^2 =$ Maximumand $U = 0$

Question169

If a spring has time period T , and is cut into n equal parts, then the time period of each part will be [2002]

Options:

A. $T\sqrt{n}$

B. $T / \sqrt{n}$

C. nT

D. T

Answer: B

Solution:

Solution:
Let k be the spring constant of the original spring.

Time period $T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mass of oscillating body.

When the spring is cut into n equal parts, the spring constant of one part becomes nk. Therefore the new time period,

$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$

Question170

A child swinging on a swing in sitting position, stands up, then the time period if the swing will [2002]

Options:

A. increase

B. decrease

C. remains same

D. increases if the child is long and decreases if the child is short

Answer: B

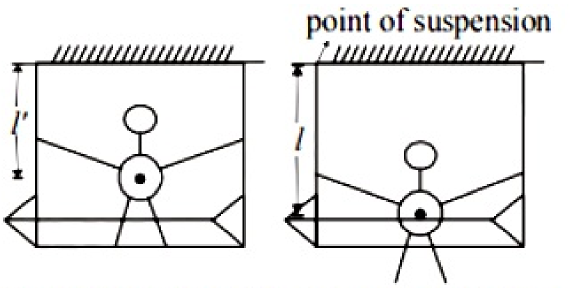
Solution:

Solution:
The time period $T = 2\pi\sqrt{\frac{l}{g}}$ where $l =$ distance between the point of suspension and the centre of mass of the child.

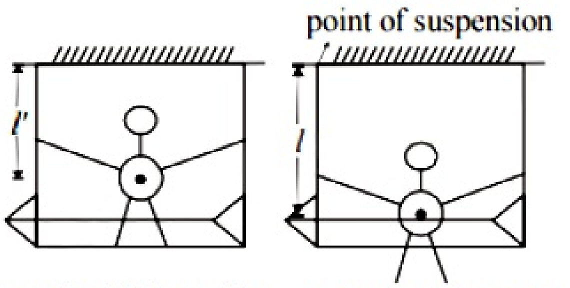
As the child stands up, her centre of mass is raised. The distance between point of suspension and centre of mass decreases ie length l decreases.

$\therefore l' < l$

$\therefore T' < T$ i.e., the period decreases.



Case (ii) child standing Case (i) child sitting



Case (ii) child standing Case (i) child sitting
